Stat 104 – Lecture 14

Binomial R. V.

- A special type of discrete random variable.
- Counts the number of "successes" in a series of independent trials.

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Example

- Draw three times, with replacement, from the bag o' chips.
- Count the number of times you win bonus points (get a blue or red chip).

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Example

- Number of independent trials
  – n = 3
- Probability of success
  – p = 0.40
- Count the number of successes
  – x = 0, 1 ,2 , or 3

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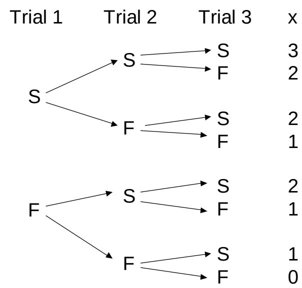
Stat 104 – Lecture 14

Tree Diagram



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Probability Distribution

- P(3) = (0.4)*(0.4)*(0.4) = 0.064
- P(2) = (0.4)*(0.4)*(0.6) +
  (0.4)*(0.6)*(0.4) +
  (0.6)*(0.4)*(0.4)
  = $3(0.4)^2(0.6)$ = 0.288

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Probability Distribution

- P(1) = (0.4)*(0.6)*(0.6) +
  (0.6)*(0.4)*(0.6) +
  (0.6)*(0.6)*(0.4)
  = $3(0.4)(0.6)^2$ = 0.432
- P(0) = (0.6)*(0.6)*(0.6) = 0.216

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Binomial R. V.

X = number of times bonus points are won on three draws from the bag o' chips.

x	0	1	2	3
P(x)	0.216	0.432	0.288	0.064

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Binomial R. V.

- General formula for the probability function.

$$P(x) = \binom{n}{x} p^x (1 - p)^{n-x} \quad x = 0,1,2,\ldots,n$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

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Example

- n = 3, p = 0.4, x = 2

$$P(2) = \binom{3}{2}(0.4)^2(0.6)^{3-2}$$

$$= \frac{3 \cdot 2 \cdot 1}{(2 \cdot 1)(1)}(0.16)(0.6)$$

$$P(2) = 0.288$$

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Binomial R. V.

- Mean

$$\mu = np$$

- Variance

$$\sigma^2 = np(1 - p)$$

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Example

- n = 3, p = 0.40, (1 – p) = 0.60
- Mean

$$\mu = np = \mathbf{3(0.4) = 1.2}$$

- Variance

$$\sigma^2 = np(1 - p)$$
$$= \mathbf{3(0.4)(0.6) = 0.72}$$

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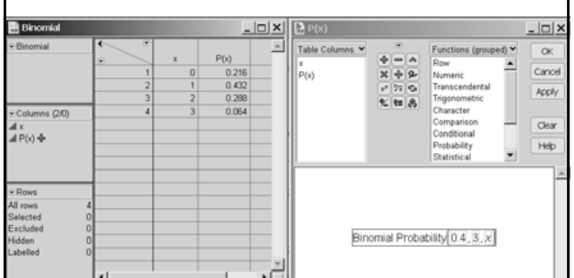
JMP



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Another Example

- 38% (p =0.38) of people in the United States have O+ blood type.
- If ten (n = 10) people come in to donate blood, what is the chance that 3 (x = 3) of them will have O+?

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Example

- n = 10, p = 0.38, x = 3

$$P(3) = \binom{10}{3}(0.38)^3(0.62)^7$$

$$= \frac{10!}{(3!)(7!)}(0.054872)(0.035216146)$$

$$P(3) = 0.2319$$

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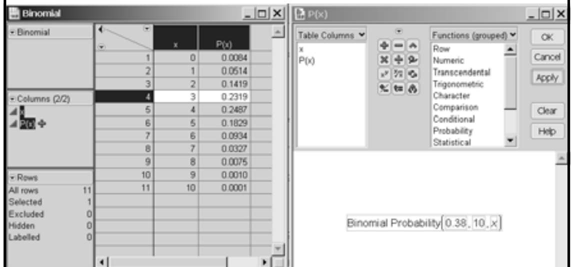
P(3) = 0.2319



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