

Stat 104 – Lecture 14

Binomial R. V.

- A special type of discrete random variable.
- Counts the number of “successes” in a series of independent trials.

1

Example

- Draw three times, with replacement, from the bag o’ chips.
- Count the number of times you win bonus points (get a blue or red chip).

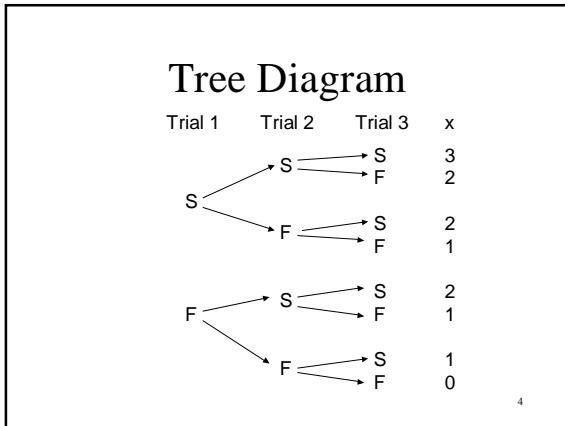
2

Example

- Number of independent trials
– $n = 3$
- Probability of success
– $p = 0.40$
- Count the number of successes
– $x = 0, 1, 2, \text{ or } 3$

3

Stat 104 – Lecture 14



Probability Distribution

- $P(3) = (0.4)*(0.4)*(0.4) = 0.064$
- $P(2) = (0.4)*(0.4)*(0.6) + (0.4)*(0.6)*(0.4) + (0.6)*(0.4)*(0.4) = 3(0.4)^2(0.6) = 0.288$

5

Probability Distribution

- $P(1) = (0.4)*(0.6)*(0.6) + (0.6)*(0.4)*(0.6) + (0.6)*(0.6)*(0.4) = 3(0.4)(0.6)^2 = 0.432$
- $P(0) = (0.6)*(0.6)*(0.6) = 0.216$

6

Stat 104 – Lecture 14

Binomial R. V.

X = number of times bonus points are won on three draws from the bag o' chips.

x	0	1	2	3
P(x)	0.216	0.432	0.288	0.064

7

Binomial R. V.

- General formula for the probability function.

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, 2, \dots, n$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

8

Example

- $n = 3, p = 0.4, x = 2$

$$\begin{aligned} P(2) &= \binom{3}{2} (0.4)^2 (0.6)^{3-2} \\ &= \frac{3 \cdot 2 \cdot 1}{(2 \cdot 1)(1)} (0.16)(0.6) \\ P(2) &= 0.288 \end{aligned}$$

9

Stat 104 – Lecture 14

Binomial R. V.

- Mean

$$\mu = np$$

- Variance

$$\sigma^2 = np(1 - p)$$

10

Example

- $n = 3, p = 0.40, (1 - p) = 0.60$

- Mean

$$\mu = np = 3(0.4) = 1.2$$

- Variance

$$\begin{aligned}\sigma^2 &= np(1 - p) \\ &= 3(0.4)(0.6) = 0.72\end{aligned}$$

11

JMP

The screenshot shows the JMP software interface. On the left, a table displays the binomial probability distribution for $n=3$ and $p=0.4$. The table has columns for x and $P(x)$.

x	$P(x)$
1	0.216
2	0.432
3	0.288
4	0.064

On the right, a dialog box titled "Functions (grouped)" is open, showing a list of functions including Row, Numeric, Transcendental, Trigonometric, Character, Comparison, Conditional, Probability, and Statistical. The "Probability" function is selected, and the input field shows "Binomial Probability(0.4, 3, x)".

12

Stat 104 – Lecture 14

Another Example

- 38% ($p = 0.38$) of people in the United States have O+ blood type.
- If ten ($n = 10$) people come in to donate blood, what is the chance that 3 ($x = 3$) of them will have O+?

13

Example

- $n = 10$, $p = 0.38$, $x = 3$

$$P(3) = \binom{10}{3} (0.38)^3 (0.62)^7$$
$$= \frac{10!}{(3!)(7!)} (0.054872)(0.035216146)$$
$$P(3) = 0.2319$$

14

$$P(3) = 0.2319$$

The screenshot shows a spreadsheet window titled 'Binomial' with columns 'x' and 'P(x)'. The data is as follows:

x	P(x)
1	0.0084
2	0.0514
3	0.1419
4	0.2319
5	0.2487
6	0.1829
7	0.0934
8	0.0327
9	0.0075
10	0.0010
11	0.0001

Overlaid on the spreadsheet is a 'Table Columns' dialog box for the function 'P(x)'. The 'Functions (grouped)' list includes: Row, Numeric, Transcendental, Trigonometric, Character, Comparison, Conditional, Probability, and Statistical. The 'Binomial Probability(0.38, 10, x)' formula is visible in the spreadsheet cell.

15
