Stat 104 – Homework 6
Solution

Assignment:

1. Complete the following problems from the text: 6.3, 6.4, 6.7, 6.8, 6.23, 6.24, 6.27, 6.28, 6.43, and 6.44.

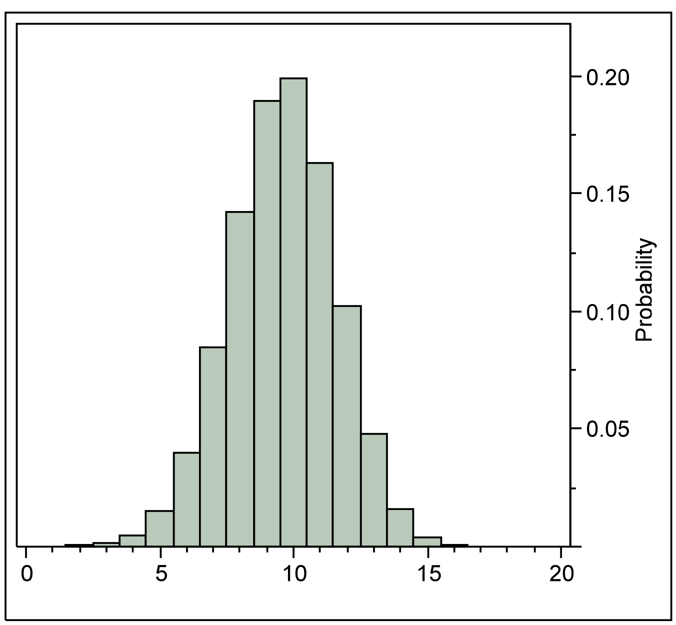
   If you have questions on these problems, please see your course instructor.

2. For ewes (female sheep), pregnancy results in one lamb 60% of the time and in multiple lambs 40% of the time. Different ewes are independent in terms of whether they will have single or multiple lambs. Suppose that there are 16 ewes about to give birth.

   a) Use JMP to calculate the probability distribution for the number of births that result in single lambs. Round probabilities to 4 decimal places.

x	P(x)	x	P(x)	x	P(x)
0	**0.0000**	**6**	**0.0392**	**12**	**0.1014**
1	**0.0000**	**7**	**0.0840**	**13**	**0.0468**
2	**0.0001**	**8**	**0.1417**	**14**	**0.0150**
3	**0.0008**	**9**	**0.1889**	**15**	**0.0030**
4	**0.0040**	**10**	**0.1983**	**16**	**0.0003**
5	**0.0142**	**11**	**0.1623**		

   b) Have JMP create a histogram of the probability distribution. Be sure the histogram has a probability axis.



c) Describe the shape of the probability distribution.

The shape is fairly symmetric with a single mound at 10. There might be a slight skew to the left.

d) What is the probability that there are exactly 10 births resulting in single lambs?

0.1983

e) What is the probability that more than 10 births result in single lambs?

Add up the probabilities for 11, 12, 13, 14, 15 and 16 births resulting in single lambs.
0.1623 + 0.1014 + 0.0468 + 0.0150 + 0.0030 + 0.0003 = 0.3288

f) What is the probability that 5 or fewer births result in single lambs?

Add up the probabilities for 0, 1, 2, 3, 4, and 5 births resulting in single lambs.
0.0000 + 0.0000 + 0.0001 + 0.0008 + 0.0040 + 0.0142 = 0.0191

g) What is the probability that 5 or fewer births result in multiple lambs?

Five or fewer births resulting in multiple lambs is the same as 11 or more births resulting in single lambs. Add up the probabilities for 11, 12, 13, 14, 15 and 16 births resulting in single lambs.
0.1623 + 0.1014 + 0.0468 + 0.0150 + 0.0030 + 0.0003 = 0.3288

h) What is the mean number of births resulting in single lambs, round to 1 decimal place? Explain how the mean number can be a fraction, even though number of births is a whole number.

$\boldsymbol{mean = np = 16(0.6) = 9.6}$.
This is an average number of births resulting in single lambs. For groups of 16 births, sometimes there will be 7 births resulting in single lambs, other times 8, other times 9, etc. So if we averaged the births resulting in single lambs over many, many groups of 16, the average number would be 9.6 births with single lambs out of 16.

i) What is the standard deviation of births resulting in single lambs, round to 2 decimal places?

$$\textbf{standard deviation} = \sigma = \sqrt{np(1-p)} = \sqrt{16(0.6)(1-0.6)} = 1.96$$

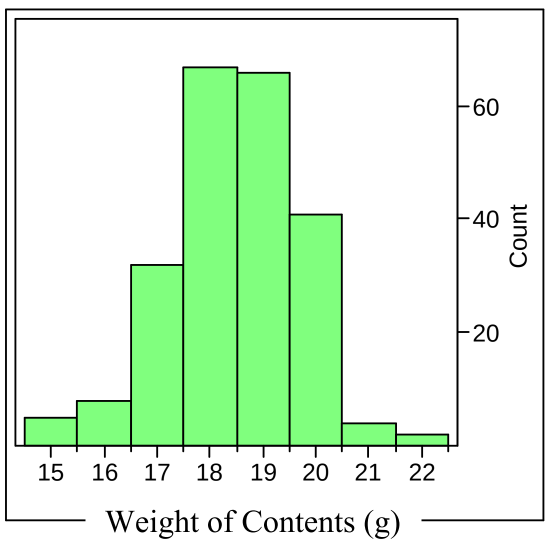
j) What is the probability that the number of births resulting in single lambs falls between the mean minus two standard deviations and the mean plus two standard deviations?

We want the probability that the number of births resulting in multiple lambs is between 9.6 – 3.9 = 5.7 and 9.6 + 3.9 = 13.5, so x = 6, 7, 8, 9, 10, 11, 12, or 13.

0.0392 + 0.0840 + 0.1417 + 0.1889 + 0.1983 + 0.1623 + 0.1014 + 0.0468 = 0.9626

k) Turn in the JMP data table that contains the binomial probabilities as well as the JMP output summarizing the distribution.

3. In the first lab data on the weights of unopened Fun Size bags of M&Ms were collected. Last year, data on the weight of contents, just the M&Ms with no bag, were collected. Below is a histogram of weights of contents of Fun Size bags.



a) Describe the shape of the histogram. Why is it reasonable to use a normal model for the distribution of the weight of contents for all Fun Size bags of M&M's?

The shape is mounded at 18.5 and symmetric.
A normal model is symmetric and mounded in the middle. Because the shape of the histogram is consistent with that of a normal model, a normal model is reasonable to use for the distribution of the net weight of all Fun Size bags of M&M's.

b) Use a normal model with μ = 18.5 g, and σ = 1.25 g for the distribution of weight of contents for Fun Size bags of M&M's.

i. What is the probability that a Fun Size bag will have a net weight less than 16 g?

$$z = \frac{y-\mu}{\sigma} = \frac{16-18.5}{1.25} = -2.00$$

From Table Z, the probability of being less than z = –2.00 is 0.0228 or 2.3%. If you use the 68 – 95 – 99.7 rule and answered 2.5%, that is also acceptable.

ii. What is the probability that a Fun Size bag will have a weight of contents greater than 22 g?

$$z = \frac{y-\mu}{\sigma} = \frac{22-18.5}{1.25} = 2.80$$

From Table Z, the probability of being less than z = 2.80 is 0.9974. Therefore the probability of being greater than z = 2.80 is 1 – 0.9974 = 0.0026 or 0.3%.

iii. What is the probability that a Fun Size bag will have a weight of contents between 17 g and 20 g?

$$z_1 = \frac{y-\mu}{\sigma} = \frac{17-18.5}{1.25} = -1.20$$

$$z_2 = \frac{y-\mu}{\sigma} = \frac{20-18.5}{1.25} = 1.20$$

From Table Z, the probability of being less than z = –1.20 is 0.1151 and the probability of being less than z = 1.20 is 0.8849. Therefore the probability of being between the two is 0.8849 – 0.1151 = 0.7698 or 77%.

iv. We wish to label the Fun Size bag such that 97% of all Fun Size bags will contain at least the labeled weight. What should the label weight be?

$$z = \frac{y-\mu}{\sigma} \Rightarrow -1.88 = \frac{\text{label wgt} - 18.5}{1.25}$$

$$\text{label wgt} = -1.88(1.25) + 18.5 = 16.15\text{ g}$$