1. [20 pts] Short answer.
   a) [3] Statistics is about … _______________. (Fill in the blank with one word.)

   b) [3] Indicate whether each variable is discrete or continuous. For continuous variables, give the appropriate units.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Discrete</th>
<th>Continuous (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pigs in a litter.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size of a field</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c) [4] 44.6% of all undergraduate students enrolled at ISU are female. 12.5% of all undergraduate students enrolled at ISU are in the College of Agriculture. 5.2% of all undergraduate students enrolled at ISU are females in the College of Agriculture. If a student is selected at random from all students enrolled at ISU what is the chance that student is a female or in the College of Agriculture?

Use the following information for the next two problems. A cow usually gives birth to a single calf. However, multiple births (e.g. twins, triplets, etc) do occur. Below is the probability distribution for the number of calves born per full term pregnancy.

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>3 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>0.75</td>
<td>0.24</td>
<td>0.01</td>
</tr>
</tbody>
</table>

d) [3] Replace “3 or more” with 3 and compute the mean number of calves born per full term pregnancy.

e) [3] Is the value computed in d) greater than, equal to, or less than the mean of the random variable X? Explain briefly.
The four graphs below depict the distribution of the sample mean for samples of size \( n \) from a population that is normally distributed with mean \( \mu = 10 \) and standard deviation \( \sigma = 10 \).

Which graph depicts the sampling distribution of the sample mean when

- \( n = 1 \)?
- \( n = 10 \)?
- \( n = 50 \)?
- \( n = 100 \)?

2. [10 pts] Students in introductory classes were asked how many brothers they have. Below is the distribution of the number of brothers. The number on top of a bar is the percentage of students with that many brothers.

a) [2] Describe the shape of the distribution.
b) [2] What is the most frequently occurring number of brothers? What percentage of students has this many brothers?

c) [2] What percentage of students has 3 or more brothers?

d) [4] The mean of this distribution is 1.077 and the standard deviation is 1.038. What is the probability that the number of brothers is within 2 standard deviations of the mean?

3. [20 pts] A study was done in Michigan with students in grades 4 – 6. The students were asked the following question: What would you most like to do at school? The choices were Make good grades, Be good at sports, or Be popular. Students came from Rural, Suburban and Urban schools. Below are the data.

<table>
<thead>
<tr>
<th></th>
<th>Make good grades.</th>
<th>Be good at sports.</th>
<th>Be popular.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td>57</td>
<td>50</td>
<td>42</td>
<td>149</td>
</tr>
<tr>
<td>Suburban</td>
<td>87</td>
<td>42</td>
<td>22</td>
<td>151</td>
</tr>
<tr>
<td>Urban</td>
<td>103</td>
<td>49</td>
<td>26</td>
<td>178</td>
</tr>
<tr>
<td>Total</td>
<td>247</td>
<td>141</td>
<td>90</td>
<td>478</td>
</tr>
</tbody>
</table>

One student is selected at random from the 478.

a) [2] What is the probability the student selected is from a rural school?

b) [2] What is the probability the student selected wants to make good grades?

c) [4] What is the probability the student selected is from a rural school or wants to make good grades?
d) [4] What is the probability the student selected wants to make good grades given the student is from a rural school?


f) [5] Are make good grades and rural school independent? Support your answer with appropriate probability calculations.

4. [20 pts] For cows, pregnancy results in one calf 75% of the time and in multiple calves 25% of the time. Different cows are independent in terms of whether they will have single or multiple calves.

a) [3] For three cows, what is the probability that none of the pregnancies result in multiple calves?

b) [4] For three cows what is the probability that exactly one pregnancy will result in multiple calves?
Below is a table of probabilities for the binomial random variable, x, the number of cows out of 20 whose pregnancy results in multiple calves.

<table>
<thead>
<tr>
<th>x</th>
<th>P(x)</th>
<th>x</th>
<th>P(x)</th>
<th>x</th>
<th>P(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0032</td>
<td>5</td>
<td>0.2023</td>
<td>10</td>
<td>0.0099</td>
</tr>
<tr>
<td>1</td>
<td>0.0211</td>
<td>6</td>
<td>0.1686</td>
<td>11</td>
<td>0.0030</td>
</tr>
<tr>
<td>2</td>
<td>0.0669</td>
<td>7</td>
<td>0.1124</td>
<td>12</td>
<td>0.0008</td>
</tr>
<tr>
<td>3</td>
<td>0.1339</td>
<td>8</td>
<td>0.0609</td>
<td>13</td>
<td>0.0002</td>
</tr>
<tr>
<td>4</td>
<td>0.1897</td>
<td>9</td>
<td>0.0271</td>
<td>14 or more</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

c) [2] Describe the shape of the distribution.

d) [3] What is the probability that 4 or fewer cows have pregnancies that result in multiple calves?

e) [2] What is the mean number of cows whose pregnancies result in multiple calves?

f) [2] What is the standard deviation for the binomial random variable, x, the number of cows out of 20 whose pregnancy results in multiple calves.

g) [4] What is the probability that exactly 4 cows have pregnancies that result in multiple calves given 4 or fewer cows have pregnancies that result in multiple calves?
5. [18 pts] When you go in for a physical exam the doctor often asks that you have blood drawn so that tests can be run on your blood. Many of these blood test results can be considered random variables that follow normal models. For example, phosphorous (a structural component of bone) follows a normal model with mean $\mu = 3.5$ mg/dl and $\sigma = 0.5$ mg/dl. You must show your work in order to earn credit.
   
a) [3] Low blood phosphorous can lead to softening of the bones. What percentage of people has phosphorous levels less than 2.6 mg/dl?

b) [5] High blood phosphorous can lead to calcification of non-skeletal tissues. What percentage of people has phosphorous levels greater than 4.6 mg/dl?

c) [5] The “normal range” for phosphorous is 2.5 to 4.5. What percentage of people has phosphorous levels within the normal range?

d) [5] A doctor wants to establish a cut-off value such that only 0.5% of phosphorous levels will be greater than that cut-off. What should the cut-off value be?
6. [12 pts] On homework 6, we used a normal model to describe the net weight of Fun Size bags of M&M’s. For this normal model the mean is 18.5 g and the standard deviation is 1.2 g. Fun Size bags are sold in larger packages of 8 randomly selected bags. We are interested in the sample mean weight of bags sold in these larger packages.

   a) [4] Describe the distribution of the sample mean weight of 8 Fun Size bags. Be sure to tell me the shape of the distribution, the center (mean) of the distribution and the spread (standard deviation) of the distribution.

   b) [4] What is the probability that the sample mean weight of 8 Fun Size bags is less than 18 g?

   c) [4] What is the probability that the sample mean weight of 8 Fun Size bags is greater than 19.4 g?
Formulas

Probability Rules:

Complements A and $A^C$: $P(A^C) = 1 - P(A)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ and } B) = P(A)P(B|A)=P(B)P(A|B)$

A and B are mutually exclusive if $P(A \text{ and } B) = 0$

A and B are independent if $P(A) = P(A|B)$ or $P(B) = P(B|A)$

Discrete random variable: $x$

Probability distribution: $0 \leq P(x) \leq 1$, $\sum P(x) = 1$

Mean: $\mu = \sum xP(x)$, Standard deviation: $\sigma = \sqrt{\sum (x - \mu)^2 P(x)}$

Binomial random variable: $x$ counts the number of successes in $n$ independent trials where the probability of success on any one trial is $p$

Probability function: $P(x) = \binom{n}{x} p^x (1 - p)^{n-x} \quad x = 0,1,2,\ldots,n$

Mean: $\mu = np$, Standard deviation: $\sigma = \sqrt{np(1 - p)}$

Normal random variable, $y$:

Mean: $\mu$, Standard deviation: $\sigma$

Standardize: $z = \frac{y - \mu}{\sigma}$

Distribution of the sample mean, $\bar{y}$:

Shape: approximately normal,

Mean: $\mu$, Standard deviation: $SD(\bar{y}) = \frac{\sigma}{\sqrt{n}}$

Standardize: $z = \frac{\bar{y} - \mu}{SD(\bar{y})}$