

Stat 104 – Lecture 7

Linear Regression

- Example: Body mass (kg) and Bite force (N) for *Canidae*.
 - y, Response: Bite force (N)
 - x, Explanatory: Body mass (kg)
 - Cases: 28 species of *Canidae*.

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Correlation Coefficient

- Body mass and Bite force

$$r = \frac{\sum z_x z_y}{n - 1} = \frac{26.4796}{27}$$

- $r = 0.9807$

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Correlation Coefficient

- There is a strong correlation, linear association, between the body mass and bite force for the various species of *Canidae*.

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Linear Model

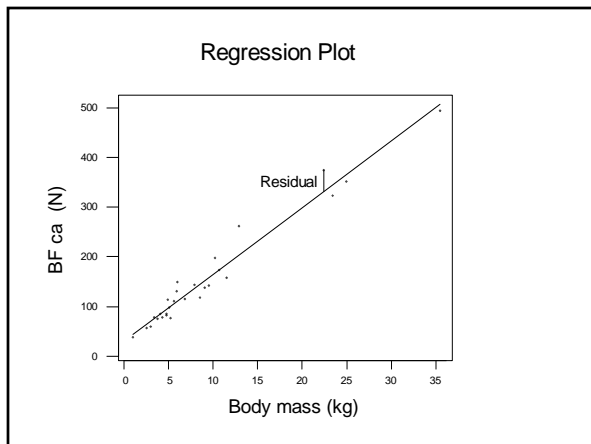
- The linear model is the equation of a straight line through the data.
- A point on the straight line through the data gives a predicted value of y , denoted \hat{y} .

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Residual

- The difference between the observed value of y and the predicted value of y , \hat{y} , is called the residual.
- Residual = $y - \hat{y}$

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Line of “Best Fit”

- There are lots of straight lines that go through the data.
- The line of “best fit” is the line for which the sum of squared residuals is the smallest – the least squares line.

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Line of “Best Fit”

- Some positive and some negative residuals but they sum to zero.
- Passes through the point (\bar{x}, \bar{y})

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Line of “Best Fit”

$$\hat{y} = a + bx$$

Least squares
slope: $b = r \frac{s_y}{s_x}$

intercept: $a = \bar{y} - b\bar{x}$

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Least Squares Estimates

Body mass, x Bite Force, y
 $\bar{x} = 9.207$ kg $\bar{y} = 154.029$ N
 $s_x = 8.016$ kg $s_y = 109.760$ N
 $r = 0.9807$

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Least Squares Estimates

$$b = 0.9807 \frac{109.760}{8.016} = 13.428$$
$$a = 154.029 - 13.428(9.207) = 30.397$$
$$\hat{y} = 30.397 + 13.428x$$

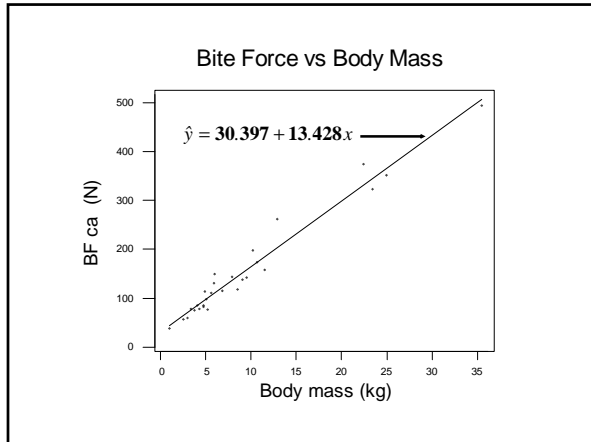
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Interpretation

- Slope – for a 1 kg increase in body mass, the bite force increases, on average, 13.428 N.
- Intercept – there is not a reasonable interpretation of the intercept in this context because one wouldn't see a *Canidae* with a body mass of 0 kg.

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Prediction

- Least squares line

$$\hat{y} = 30.397 + 13.428x$$

$$x = 25$$

$$\hat{y} = 30.397 + 13.428(25) = 366.1 \text{ N}$$

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Residual

- Body mass, $x = 25$ kg
- Bite force, $y = 351.5$ N
- Predicted, $\hat{y} = 366.1$ N
- Residual, $y - \hat{y} = 351.5 - 366.1$
 $= -14.6$ N

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