t Distribution

Confidence Level

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>80%</th>
<th>90%</th>
<th>95%</th>
<th>98%</th>
<th>99%</th>
<th>99.8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>df</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
| 9                | 2.262

95% Confidence Interval

• $\bar{Y} = 4.762$
• $n = 10$, $df = (10 - 1) = 9$
• $s = 0.314$
• $t^* = 2.262$

Calculation

$\bar{Y} - t^* \left( \frac{s}{\sqrt{n}} \right)$ to $\bar{Y} + t^* \left( \frac{s}{\sqrt{n}} \right)$

$4.762 - 2.262 \left( \frac{0.314}{\sqrt{10}} \right)$ to $4.762 + 2.262 \left( \frac{0.314}{\sqrt{10}} \right)$

$4.762 - 0.225$ to $4.762 + 0.225$

$4.537$ to $4.987$
Interpretation

• We are 95% confident that the population mean alcohol content of beer is between 4.537% and 4.987%.

Interpretation

• The population mean alcohol content of beer could be any value between 4.537% and 4.987%.
• If we repeat the procedure that produces a confidence interval, 95% of intervals produced will capture the population mean.

Testing Hypotheses

• Use sample data, in the form of a sample statistic, to support or refute hypotheses.
• How likely is it to get the sample data if a hypothesis is true?
Testing – Step by Step
- Step 1 – Hypothesis
- Step 2 – Conditions
- Step 3 – Test Statistic
- Step 4 – P-value
- Step 5 – Results

Step 1 – Hypotheses
- $\mu$ is the population mean alcohol content of beer.
- Null hypothesis
  - $H_0: \mu = 5$
- Alternative hypothesis
  - $H_A: \mu < 5$

Step 2 – Conditions
- Quantitative variable – alcohol content
- Randomization – random sample of 10 beers
- Distribution of alcohol content is approximately normal.
Step 3 – Sample Evidence
• Sample mean, 
  \( \bar{y} = 4.762 \)
• Sample standard deviation, 
  \( s = 0.314 \)

Step 3 – Sample Evidence
• Test statistic, 
\[
t = \frac{\bar{y} - \mu_0}{\left( \frac{s}{\sqrt{n}} \right)} = \frac{4.762 - 5}{\left( \frac{0.314}{\sqrt{10}} \right)} = -0.238 \approx -2.397
\]
• Even though \( t \) is negative look up the positive value.

Table T

<table>
<thead>
<tr>
<th>df</th>
<th>0.100</th>
<th>0.050</th>
<th>0.025</th>
<th>P-value</th>
<th>0.010</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2.262</td>
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<td>2</td>
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<td>2.397</td>
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<tr>
<td>3</td>
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<td>2.821</td>
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</tbody>
</table>

The P-value is between 0.01 and 0.025
Step 4 – Probability Value

- Alternative hypothesis is one sided.
  - $H_A: \mu < 5$
- P-value is the right tail probability, $df = 9$.
- Table T: P-value is between 0.01 and 0.025.

Step 5 - Results

- Reject the null hypothesis because the P-value is small, less than 0.05
- The population mean alcohol content of beer is less than 5%.

Confidence Interval

- Results of the test agree with the confidence interval.
- We are 95% confident that the population mean alcohol content of beer is between 4.537% and 4.987%.