Chapters 8 and 9

Quantitative variable
Population Parameters: \( \mu \)

Population

Inference

Sample

Sample Mean

Example

• What is the mean alcohol content of beer?
• A random sample of 10 beers is taken and the alcohol content (%) is measured.

• Population – all beers.
• Variable – alcohol content, %.
• Parameter – mean alcohol content of beer.
Sample Data – Alcohol (%)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Alcohol (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molson Canadian Dark</td>
<td>5.19</td>
</tr>
<tr>
<td>Michelob Dark</td>
<td>4.76</td>
</tr>
<tr>
<td>Big Barrel Lager</td>
<td>4.32</td>
</tr>
<tr>
<td>Hamm’s</td>
<td>4.53</td>
</tr>
<tr>
<td>Tsingtao</td>
<td>4.79</td>
</tr>
<tr>
<td>Heineken Dark</td>
<td>5.17</td>
</tr>
<tr>
<td>O’Keefe Canadian</td>
<td>4.96</td>
</tr>
<tr>
<td>Olympia Lager</td>
<td>4.78</td>
</tr>
<tr>
<td>Miller Draft</td>
<td>4.85</td>
</tr>
<tr>
<td>Guinness Stout</td>
<td>4.27</td>
</tr>
</tbody>
</table>

Sample Summary

- Sample size: $n = 10$
- Sample mean: $\bar{y} = 4.762$
- Sample standard deviation: $s = 0.314$

Sampling Distribution of $\bar{y}$
Summary

• Sampling from a population that follows a Normal Model.

• Distribution of the sample mean, $\bar{y}$
  – Shape: Normal model
  – Center: $\mu$
  – Spread: $SD(\bar{y}) = \frac{\sigma}{\sqrt{n}}$

Unknown, $\sigma$

• If we do not know the value of the population standard deviation we cannot standardize and cannot use table Z.

Unknown, $\sigma$

• We can use the sample standard deviation, $s$, as an estimate of the population standard deviation, $\sigma$. 
Unknown, $\sigma$

- We can **NOT** continue to use the standard normal distribution or Table Z.
- Why?
95% Confidence?

- Simulation illustrating repeating the procedure.

Quantitative Variable

- Confidence Interval for $\mu$.

$$\bar{y} - t^* \left( \frac{s}{\sqrt{n}} \right) \text{ to } \bar{y} + t^* \left( \frac{s}{\sqrt{n}} \right)$$

- $t^*$ found in Table T, df = n – 1
Quantitative variable

- Test statistic.

\[ t = \frac{\bar{y} - \mu}{\frac{s}{\sqrt{n}}} \], Table T ⇒ P-value

Confidence Interval for \( \mu \)

\[ \bar{y} - t^* \left( \frac{s}{\sqrt{n}} \right) \text{ to } \bar{y} + t^* \left( \frac{s}{\sqrt{n}} \right) \]

\( df = n - 1 \)

Inference for \( \mu \)

- Do NOT use Table Z!

\[ \text{Table Z} \]

- Use Table T instead!