Chapters 8 and 9

Population Parameters: \( \mu \), \( \mu_y \)

Inference

Example

• What is the mean alcohol content of beer?
• A random sample of 10 beers is taken and the alcohol content (%) is measured.

• Population – all beers.
• Variable – alcohol content, %.
• Parameter – mean alcohol content of beer.
Sample Data – Alcohol (%)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Alcohol (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molson Canadian</td>
<td>5.19</td>
</tr>
<tr>
<td>Michelob Dark</td>
<td>4.76</td>
</tr>
<tr>
<td>Big Barrel Lager</td>
<td>4.32</td>
</tr>
<tr>
<td>Hamm’s</td>
<td>4.53</td>
</tr>
<tr>
<td>Tsingtao</td>
<td>4.79</td>
</tr>
<tr>
<td>Guinness Stout</td>
<td>4.27</td>
</tr>
<tr>
<td>O’Keefe Canadian</td>
<td>4.96</td>
</tr>
<tr>
<td>Olympia Lager</td>
<td>4.78</td>
</tr>
<tr>
<td>Miller Draft</td>
<td>4.85</td>
</tr>
<tr>
<td>O’Keefe Canadian</td>
<td>5.17</td>
</tr>
<tr>
<td>Michelob Dark</td>
<td>5.17</td>
</tr>
<tr>
<td>Big Barrel Lager</td>
<td>5.19</td>
</tr>
<tr>
<td>Hamm’s</td>
<td>5.17</td>
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<tr>
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<td>5.19</td>
</tr>
<tr>
<td>O’Keefe Canadian</td>
<td>5.19</td>
</tr>
<tr>
<td>Michelob Dark</td>
<td>5.19</td>
</tr>
</tbody>
</table>

Sample Summary

- Sample size: n = 10
- Sample mean: \( \bar{y} = 4.762 \)
- Sample standard deviation: s = 0.314

Sampling Distribution of \( \bar{y} \)
Summary

• Sampling from a population that follows a Normal Model.
• Distribution of the sample mean, \( \bar{y} \)
  – Shape: Normal model
  – Center: \( \mu \)
  – Spread: \( \text{SD}(\bar{y}) = \frac{\sigma}{\sqrt{n}} \)

Unknown, \( \sigma \)

• If we do not know the value of the population standard deviation we cannot standardize and cannot use table Z.

Unknown, \( \sigma \)

• We can use the sample standard deviation, \( s \), as an estimate of the population standard deviation, \( \sigma \).
Unknown, $\sigma$

- We can **NOT** continue to use the standard normal distribution or Table Z.
- Why?
95% Confidence?

- Simulation illustrating repeating the procedure.
- \[http://statweb.calpoly.edu/chance/applets/ConfSim/ConfSim.html\]

Quantitative Variable

- Confidence Interval for \( \mu \).

\[
\bar{y} - t^{*}\left(\frac{s}{\sqrt{n}}\right) \quad \text{to} \quad \bar{y} + t^{*}\left(\frac{s}{\sqrt{n}}\right)
\]

- \( t^{*} \) found in Table T, df = n – 1
Quantitative variable

- Test statistic.
  
  \[ t = \frac{\bar{y} - \mu}{\frac{s}{\sqrt{n}}}, \text{Table T } \Rightarrow \text{P-value} \]

Confidence Interval for \( \mu \)

\[ \bar{y} - t \cdot \left( \frac{s}{\sqrt{n}} \right) \text{ to } \bar{y} + t \cdot \left( \frac{s}{\sqrt{n}} \right) \]
\[ df = n - 1 \]

Inference for \( \mu \)

- Do NOT use Table Z!
- Use Table T instead!