Stat 104 – Lecture 22

Margin of Error

$$ME = z^* SE(\hat{p}) = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

The Margin of Error is the furthest $\hat{p}$ can be from p, for a given confidence.

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Margin of Error

$$ME = z^* SE(\hat{p}) = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Confidence	80%	90%	95%	98%	99%
z*	1.282	1.645	2 or 1.96	2.326	2.576

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Another Example

- A USA Today/Gallup Poll conducted October 16–19, 2009 asked 1,521 adults nationwide the following question:

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Another Example

- "Which of the following should be Barack Obama's top priority as president: the economy, health care, the situations in Iraq and Afghanistan, energy, the federal budget deficit, or something else?"

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Another Example

- 64% of the sample said that the economy should be the top priority.
- Construct a 98% confidence interval for the population proportion who would say the economy should be the top priority.

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Another Example

$$\hat{p} = 0.64$$

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.64(0.36)}{1521}} = 0.012$$

$$0.64 - 2.326(0.012) \text{ to } 0.64 + 2.326(0.012)$$

$$0.64 - 0.028 \text{ to } 0.64 + 0.028$$

$$0.612 \text{ to } 0.668$$

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Interpretation

- We are 98% confident that between 61.2% and 66.8% of all adults would say that the economy should be the top priority.

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Sample Size

- General formula for sample size.

$$n = \frac{(z^*)^2 \hat{p}(1-\hat{p})}{ME^2}$$

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Sample Size

- Priority example. ME = 0.01 with 98% confidence.

$$\boldsymbol{n = \frac{(z^*)^2 \hat{p}(1-\hat{p})}{ME^2} = \frac{(2.326)^2(0.64)(0.36)}{(0.01)^2}}$$

$$\boldsymbol{n = 12{,}466}$$

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Inference: Hypothesis Test

- Propose a value for the population proportion, *p*.
- Does the sample data support this value?

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Example

- A seed company claims that it's new corn hybrid has a 94% germination rate.
- For a random sample of 500 seeds only 452 germinate.
- What do you think of the company's claim?

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Example

- Population: All seeds for the new corn hybrid.
- Parameter: Proportion of all seeds for the new corn hybrid that will germinate, *p*.

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Example

- Null Hypothesis
  - H_0: $p = 0.94$
- Alternative Hypothesis
  - H_A: $p < 0.94$

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Example

- How likely is it to get a sample proportion as extreme as the one we observe when taking a random sample of 500 from a population with $p = 0.94$?

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Example

- Sampling distribution of $\hat{p}$
  - Shape approximately normal.
  - Mean: $p = 0.94$
  - Standard Deviation:

$$\sqrt{\frac{0.94(0.06)}{500}} = 0.0106$$

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Standardize

$$\hat{p} = \frac{452}{500} = 0.904$$

$$z = \frac{0.904 - 0.94}{\sqrt{\frac{0.94(0.06)}{500}}} = \frac{-0.036}{0.0106} = -3.40$$

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Use Table Z

z	.00	.01	.02
–3.4	**.0003**		
–3.3			
–3.2			

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Interpretation

- Getting a sample proportion of 0.904 or less will happen only 0.03% (P-value = 0.0003) of the time when taking random samples of 500 from a population with population proportion *p*=0.94.

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