Probability Distributions

• Random variable
  – Numerical values associated with the outcomes of a random phenomenon.

Probability Distributions

• Discrete random variable
  – Numerical values associated with a distinct (discrete) set of points on the number line.
  – The probability distribution assigns a probability to each numerical value.

The Deal - Continued

• Bag o’ chips (poker chips).
  – Some are red.
  – Some are white.
  – Some are blue.
• Draw a chip from the bag.
The Deal - Continued

- Draw a red chip win 3 bonus points.
- Draw a blue chip win 1 bonus points.
- Draw a white chip lose 1 bonus points.

Discrete Random Variable

- $X =$ number of bonus points

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-1$</th>
<th>$+1$</th>
<th>$+3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>0.60</td>
<td>0.30</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Discrete R.V.

- $X =$ number of bonus points

![Bar chart showing probabilities of $X$ ranging from $-1$ to $+3$.]
Discrete R.V.

- Property 1
  \[ 0 \leq P(x) \leq 1 \]
- Property 2
  \[ \sum P(x) = 1 \]

Mean of a Discrete R.V.

- The “center” of the distribution of values found as a weighted average of the values.

\[
\mu = \sum [xP(x)]
\]

Discrete Random Variable

- \( X = \) number of bonus points

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>+1</th>
<th>+3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) )</td>
<td>0.60</td>
<td>0.30</td>
<td>0.10</td>
</tr>
<tr>
<td>( xP(x) )</td>
<td>-0.60</td>
<td>+0.30</td>
<td>+0.30</td>
</tr>
</tbody>
</table>
Mean of a Discrete R.V.

- \( X = \) number of bonus points

### Rake it in!™

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0</td>
<td>( 1,115,988/1,440,000 )</td>
</tr>
<tr>
<td>$1</td>
<td>( 192,000/1,440,000 )</td>
</tr>
<tr>
<td>$2</td>
<td>( 72,000/1,440,000 )</td>
</tr>
<tr>
<td>$3</td>
<td>( 28,800/1,440,000 )</td>
</tr>
<tr>
<td>$7</td>
<td>( 19,200/1,440,000 )</td>
</tr>
<tr>
<td>$14</td>
<td>( 7,200/1,440,000 )</td>
</tr>
<tr>
<td>$31</td>
<td>( 4,800/1,440,000 )</td>
</tr>
<tr>
<td>$1,500</td>
<td>( 12/1,440,000 )</td>
</tr>
</tbody>
</table>

If all 1,440,000 tickets are sold and if all prizes are claimed, the Iowa Lottery will payout $824,400.

- Mean payout
  - \( \mu = \frac{824,400}{1,440,000} = \$0.5725 \)
  - This means the Iowa Lottery pays out, on average, under 60 cents for every $1 ticket sold.