

## Statistics 101 – Homework 9

### Solution

1. Many people are superstitious about the number 13. Friday the 13<sup>th</sup> occurred in April and July of 2007 and will occur in June of this year. But does triskaidekaphobia (fear of the number 13) have economic implications for large high-rise hotels? A *USA Today*/Gallup poll conducted February 9-11, 2007 asked 1,006 randomly selected people 18 years old and older in telephone interviews “Suppose you checked into a hotel and were given a room on the thirteenth floor. Would this bother you or not?” 87% of the survey respondents answered they would not be bothered while 13% answered that they would be bothered.

- a) What is the population? Be specific.

**The population consists of people 18 years old and older with a telephone.**

- b) What is the sample? Be specific.

**The sample consists of the 1,006 randomly selected people.**

- c) Are the conditions for constructing a confidence interval for the population proportion satisfied? Support your answer by verifying the appropriate conditions.

**Randomization condition: People were randomly selected so this condition is met.**

**10% condition: 1,006 is less than 10% of all people 18 years old and older with telephones. This condition is met.**

**Success/Failure condition:**  $n\hat{p} = 1006(0.13) = 131$  **both of these are**  
 $n(1 - \hat{p}) = 1006(0.87) = 875$   
**greater than 10 so this condition is met.**

- d) Construct a 95% confidence interval for the population proportion of people 18 years and older who would be bothered by being given a room on the thirteenth floor.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$0.13 \pm 2 \sqrt{\frac{0.13(0.87)}{1006}}$$

$$0.13 \pm 2(0.0106) \Rightarrow 0.13 \pm 0.02$$

$$0.11 \text{ to } 0.15$$

**You can also use  $z^*=1.96$  but the final interval will not change.**

- e) How would a 90% confidence interval compare to the one you constructed in d)? You should comment on the centers of the intervals and the widths of the intervals. Note: You do not need to construct a 90% confidence interval to answer this question.

**A 90% confidence interval would have the same center (0.13) but it would be narrower than the 95% confidence interval.**

2. In a *Gallup* Poll conducted March 6-9, 2008 1,012 adults nationwide were asked, “**With which one of these statements about the environment and the economy do you most agree? Protection of the environment should be given priority, even at the risk of curbing economic growth. OR, Economic growth should be given priority, even if the environment suffers to some extent.**” 49% of those surveyed thought the environment should be given priority. 42% of those surveyed thought the economy should be given priority, 5% of those surveyed thought they should get equal priority and 3% were unsure.

- a) What is the population?

**The population consists of all adults in the nation.**

- b) What is the sample?

**The sample consists of the 1,012 adults in the *Gallup* Poll.**

- c) Give the number, not a proportion or a percentage, of adults in the sample who thought the environment should be given priority.

**0.49\*1012 = 496 people thought the environment should be a priority.**

- d) Construct a 98% confidence interval for the proportion of the population who think the environment should be given priority.

$$\hat{p} \pm z * \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$0.49 \pm 2.326 \sqrt{\frac{0.49(0.51)}{1012}}$$

$$0.49 \pm 2.326(0.0157) \Rightarrow 0.13 \pm 0.037$$

$$0.453 \text{ to } 0.527$$

- e) Give an interpretation of your confidence interval in d).

**We are 95% confident that the proportion of the population who think the environment should be given priority is somewhere between 0.453 and 0.527.**

- f) Based on your interval in d) is the proportion of the population who think the environment should be given priority 55%? Explain briefly.

**No. The proportion, 0.55, does not fall within the confidence interval so it is not a plausible value for the population proportion.**

3. A large accounting firm recently selected 100 individuals to participate in a management-training program. Of these, 75 were males and 25 were females. The company claims that it selected the 100 individuals at random from a pool of eligible employees that consisted of 800 females and 1200 males. Does the sample agree with the company's claim or do females make up a significantly smaller proportion of those selected for the training program than the proportion of eligible females? To answer this question you should:

- a) Indicate what the population is.

**The population consists of the pool of eligible employees (800 females and 1200 males).**

- b) Indicate what the sample is.

**The sample consists of the 75 males and 25 females selected for the management-training program.**

- c) Set up a null hypothesis that the population proportion of eligible employees that are female is 40% against an alternative that the population proportion is less than 40%.

**$H_0: p = 0.40$**

**$H_A: p < 0.40$**

- d) Verify that the 10% condition and success/failure conditions are satisfied.

**10% condition is satisfied because 100 (the sample size) is less than 10% of the population size (10% of 2000 is 200).**

**The success/failure condition is satisfied because**

$$np = 100(0.40) = 40$$

$$n(1 - p) = 100(1 - 0.40) = 60$$

**are both greater than 10.**

- e) Calculate the value of the test statistic and use Table Z to find the P-value.

$$\hat{p} = \frac{25}{100} = 0.25$$

$$z = \frac{0.25 - 0.40}{\sqrt{\frac{0.40(0.60)}{100}}} = \frac{-0.15}{0.049} = -3.06$$

Table Z : P - value = 0.0011

- f) Use the P-value to make a decision whether or not to reject the null hypothesis.

**The P-value is very small therefore we should reject the null hypothesis.**

- g) State a conclusion, within the context of the problem, which addresses the company's claim.

**The proportion of eligible employees that are female is less than 40%. The sample proportion of females selected for the training program is only 25% and this is convincing evidence that females make up a significantly smaller proportion of those selected for the training program than the proportion of eligible females.**

4. A large manufacturer that sells consumer products on-line wishes to publicize its customer satisfaction in an advertisement. Specifically, it wants to state that over 90% of the manufacturer's customers would tell a friend to buy a product from the manufacturer. The manufacturer selects a random sample of 1,000 customers from its database of over 2 million customers, contacts them via email and asks them the question "Would you tell a friend to buy a product from us?" 922 say yes and 78 say no.

- a) What is the population?

**The population consists of the over 2 million customers of the manufacturer.**

- b) What is the sample?

**The sample consists of the 1,000 customers selected at random.**

- c) Give a null and alternative hypothesis for the proportion of all customers who would tell a friend to buy a product from the manufacturer.

**$H_0: p = 0.90$**

**$H_A: p > 0.90$**

- d) Verify that the 10% condition and success/failure condition are satisfied.

**The 10% condition is satisfied because 1,000 (the sample size) is less than 10% of the population size (10% of over 2 million is over 200,000).**

**The success failure condition is satisfied because**

$$np = 1000(0.90) = 900$$

$$n(1 - p) = 1000(1 - 0.90) = 100$$

**are both greater than 10.**

- e) Compute the value of the test statistic and convert this to a P-value.

$$\hat{p} = \frac{922}{1000} = 0.922$$

$$z = \frac{0.922 - 0.900}{\sqrt{\frac{0.90(0.10)}{1000}}} = \frac{0.022}{0.0095} = 2.32$$

Table Z: P - value = 1 - 0.9898 = 0.0102

- f) Use the P-value to make a decision whether or not to reject the null hypothesis.

**Reject the null hypothesis because the P-value is small (< 0.05).**

- g) State a conclusion, within the context of the problem, which addresses whether or not it is appropriate for the manufacturer to make the claim of over 90% satisfaction in its advertisement.

**The sample proportion of 93% is strong evidence that the population proportion of customers who would tell a friend to buy a product from the manufacturer is greater than 90%. The manufacturer can make the claim of over 90% satisfaction.**

5. The following question was asked as part of a standardized statistics test given to over 50,000 introductory statistics students.

True or False: The P-value is the probability that the null hypothesis is true.

For a random sample of 400 students who took the test, 176 answered true and 224 answered false. Are students guessing at the answer or not? You should support your answer in two ways.

**Note: If students are guessing then the chance that they would choose True (or False) is 50%.**

- a) By testing an appropriate hypothesis. Be sure to include all the steps and say how the test of hypothesis supports your answer.

**$H_0: p = 0.50$**

**$H_A: p \neq 0.50$**

**p is the population proportion of students who answer True.**

**Random Sample Condition: Met. The problem states there is a random sample of 1000 students.**

**10% Condition: Met. There are over 50,000 students who took the test so 400 is less than 10% of the population size.**

**Success/Failure Condition: Met.  $np=200$  and  $n(1-p)=200$  both of which are greater than 10.**

$$\hat{p} = \frac{176}{400} = 0.44$$

$$z = \frac{0.44 - 0.50}{\sqrt{\frac{0.50(0.50)}{400}}} = \frac{-0.06}{0.025} = -2.40$$

Table Z : P - value =  $2(0.0082) = 0.0164$

**The P-value is small therefore we should reject the null hypothesis.**

**The students are not guessing, picking True at random.**

- b) By constructing a 95% confidence interval. Be sure to include all the steps and say how the confidence interval supports your answer.

**Random Sample Condition: Met. The problem states there is a random sample of 1000 students.**

**10% Condition: Met. There are tens of thousands of students taking introductory statistics so 400 is less than 10% of the population size.**

**Success/Failure Condition: Met.**  $n\hat{p} = 176$   $n(1 - \hat{p}) = 224$  both of which are greater than 10.

$$\hat{p} - z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \text{ to } \hat{p} + z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$0.44 - 1.96 \sqrt{\frac{0.44(0.56)}{400}} \text{ to } 0.44 + 1.96 \sqrt{\frac{0.44(0.56)}{400}}$$

$$0.44 - 0.049 \text{ to } 0.44 + 0.049$$

$$0.391 \text{ to } 0.489$$

**Because 0.50 is not in the interval, the population proportion could not be 50%. The students are not guessing.**