

Statistics 101 – Homework 8

Solution

1. The maker of M&M's says on its website that 16% of Dark Chocolate M&M's are orange. Suppose that M&M's are packaged at random. We wish to examine the sample proportion of orange Dark Chocolate M&M's, \hat{p} , in various sized bags.

- a) For each of the different sized bags, give the mean and standard deviation of the sampling distribution of \hat{p} . Also comment on whether or not the success/failure condition is met for the sampling distribution to be approximately normal.

- i) Fun size bags containing 25 Dark Chocolate M&M's.

Distribution of \hat{p} :

$$\text{mean} = 0.16, \text{ standard deviation} = \sqrt{\frac{0.16(0.84)}{25}} = 0.073$$

Success/Failure condition: This is not met. $np = 4$ which is less than 10.

- ii) Small bags containing 50 Dark Chocolate M&M's.

Distribution of \hat{p} :

$$\text{mean} = 0.16, \text{ standard deviation} = \sqrt{\frac{0.16(0.84)}{50}} = 0.052$$

Success/Failure condition: This is not met. $np = 8$ which is less than 10.

- iii) Medium bags containing 100 Dark Chocolate M&M's.

Distribution of \hat{p} :

$$\text{mean} = 0.16, \text{ standard deviation} = \sqrt{\frac{0.16(0.84)}{100}} = 0.037$$

Success/Failure condition: This is met. $np = 16$ and $n(1-p)=84$ which are both greater than 10.

- iv) Extra large bags containing 500 Dark Chocolate M&M's

Distribution of \hat{p} :

$$\text{mean} = 0.16, \text{ standard deviation} = \sqrt{\frac{0.16(0.84)}{500}} = 0.016$$

Success/Failure condition: This is met. $np = 80$ and $n(1-p)=420$ which are both greater than 10.

- b) For the extra large bags containing 500 Dark Chocolate M&M's, use the 68-95-99.7 Rule to describe how the sample proportion of orange Dark Chocolate M&M's might vary from bag to bag.

68% of the time the sample proportion of yellow will fall between

$$0.16 - 0.016 = 0.144 \text{ and } 0.16 + 0.016 = 0.176$$

95% of the time the sample proportion of yellow will fall between

$$0.16 - 2(0.016) = 0.128 \text{ and } 0.16 + 2(0.016) = 0.192$$

99.7% of the time the sample proportion of yellow will fall between

$$0.16 - 3(0.016) = 0.112 \text{ and } 0.16 + 3(0.016) = 0.208$$

- c) In an extra large bag of 500 Dark Chocolate M&M's there are 90 orange. Is this an unusually large number of orange? Explain.

$$\hat{p} = \frac{90}{500} = 0.18 \quad z = \frac{0.18 - 0.16}{0.016} = 1.25 \quad \text{Table Z: } 0.8944$$

$$1 - 0.8944 = 0.1056$$

The chance of getting 90 or more orange Dark Chocolate M&M's in a random sample of 500 is over 10%. This is not unlikely, therefore 90 (out of 500) orange M&M's is not an unusually large number.

2. It is believed that 44% of all college students in the United States engage in binge drinking (5 or more drinks at a sitting for men, 4 or more for women). Consider a random sample of 100 college students. Verify that the success/failure condition is met. Use the 68-95-99.7 Rule to describe the sampling distribution model for the sample proportion of students who engage in binge drinking.

Success/Failure condition: Both $np = 44$ and $n(1-p) = 56$ are at least 10. This verifies that this condition is met.

Distribution of \hat{p} :

$$\text{mean} = 0.44, \text{ standard deviation} = \sqrt{\frac{0.44(0.56)}{100}} = 0.05$$

68% of the time the sample proportion of binge drinkers will fall between

$$0.44 - 0.05 = 0.39 \text{ and } 0.44 + 0.05 = 0.49$$

95% of the time the sample proportion of binge drinkers will fall between

$$0.44 - 2(0.05) = 0.34 \text{ and } 0.44 + 2(0.05) = 0.54$$

99.7% of the time the sample proportion of binge drinkers will fall between

$$0.44 - 3(0.05) = 0.29 \text{ and } 0.44 + 3(0.05) = 0.59$$

3. In 2004, 20.9% of all adults (18 years old or older) in the United States were current smokers. There were approximately 220,000,000 adults in the United States in 2004. For a random sample of 1000 U.S. adults is the 10% condition met? Explain briefly. Is the success/failure condition met? Explain briefly. Use the 68-95-99.7 Rule to describe the sampling distribution model for the proportion current smokers in a random sample of 1000 adults in the United States in 2004.

10% condition: This is met. 1000 is much less than 10% of 220,000,000.

Success/Failure condition: This is met. $np = 209$ and $n(1-p) = 791$ both of which are at least 10.

Distribution of \hat{p} :

$$\text{mean} = 0.209, \text{ standard deviation} = \sqrt{\frac{0.209(0.791)}{1000}} = 0.013$$

68% of the time the sample proportion of smokers will fall between

$$0.209 - 0.013 = 0.196 \text{ and } 0.209 + 0.013 = 0.222$$

95% of the time the sample proportion of smokers will fall between

$$0.209 - 2(0.013) = 0.183 \text{ and } 0.209 + 2(0.013) = 0.235$$

99.7% of the time the sample proportion of smokers will fall between

$$0.209 - 3(0.013) = 0.170 \text{ and } 0.209 + 3(0.013) = 0.248$$

4. A seed corn distributor advertises a germination rate of 90%. What is the probability that out of 400 randomly selected corn seeds from the distributor, fewer than 340 will germinate? Verify that the appropriate conditions for computing this probability are met? Explain.

Distribution of \hat{p} :

$$\text{mean} = 0.90, \text{ standard deviation} = \sqrt{\frac{0.90(0.10)}{400}} = 0.015$$

$$\hat{p} = \frac{340}{400} = 0.85, \quad z = \frac{0.85 - 0.90}{0.015} = -3.33$$

Table Z : 0.0004 or about a 0.04% chance

10% condition: This is met. 400 seeds is much less than 10% of all corn seeds from the distributor.

Success/Failure condition: This is met. $np = 360$ and $n(1-p) = 40$ both of which are at least 10.