Stat 101L: Lecture 33

Inference for μ

- Who? Young adults.
- What? Heart rate (beats per minute).
- When?
- Where? In a physiology lab.
- How? Take pulse at wrist for one minute.
- Why? Part of an evaluation of general health.

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Inference for μ

- What is the mean heart rate for all young adults?
- Use the sample mean heart rate, $\bar{y}$, to make inferences about the population mean heart rate, μ.

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Inference for μ

- Distribution of $\bar{y}$
  - Shape: Approximately normal
  - Center: Mean, μ
  - Spread: Standard Deviation,

$$\mathrm{SD}(\bar{y}) = \frac{\sigma}{\sqrt{n}}$$

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Problem

- The population standard σ deviation, is unknown.
- Therefore, $\mathrm{SD}(\bar{y}) = \frac{\sigma}{\sqrt{n}}$ is unknown as well.

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Solution

- Use the sample standard deviation, s to get the standard error of $\bar{y}$

$$\mathrm{SE}(\bar{y}) = \frac{s}{\sqrt{n}}$$

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Problem

- The distribution of the standardized sample mean

$$\frac{\bar{y} - \mu}{\mathrm{SE}(\bar{y})}$$

does not follow a normal model.

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Solution

- The distribution of the standardized sample mean

$$\frac{\bar{y} - \mu}{\mathrm{SE}(\bar{y})}$$

does follow a Student's t-model with df = $n - 1$.

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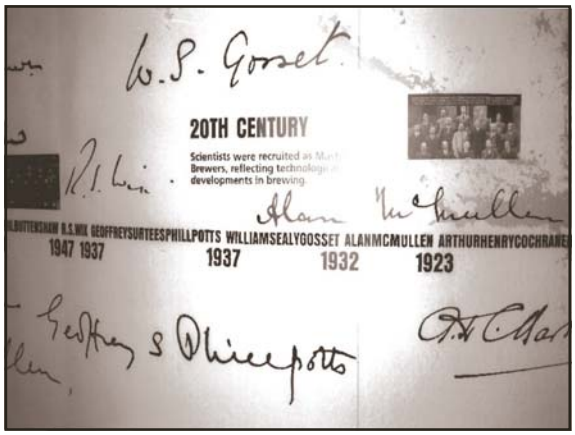

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Inference for μ

- Do NOT use Table Z!

- Use Table T instead!

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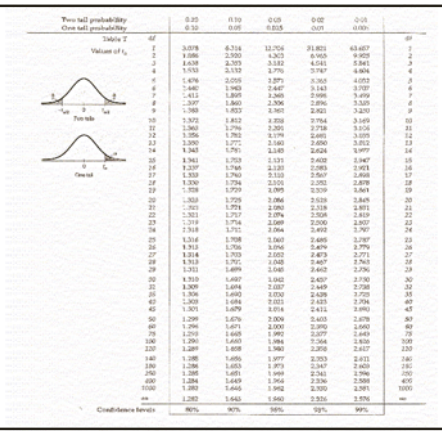

Two tail probability		0.20	0.10	0.05	0.02	0.01	0.001	
One tail probability		0.10	0.05	0.025	0.01	0.005		
Table T	df							df
Values of t_p	1	3.078	6.314	12.706	31.821	63.657		1
	2	1.886	2.920	4.303	6.965	9.925		2
	3	1.638	2.353	3.182	4.541	5.841		3
	4	1.533	2.132	2.776	3.747	4.604		4
	5	1.476	2.015	2.571	3.365	4.032		5
	6	1.440	1.943	2.447	3.143	3.707		6
	7	1.415	1.895	2.365	2.998	3.499		7
	8	1.397	1.860	2.306	2.896	3.355		8
	9	1.383	1.833	2.262	2.821	3.250		9
	10	1.372	1.812	2.228	2.764	3.169		10
	11	1.363	1.796	2.201	2.718	3.106		11
	12	1.356	1.782	2.179	2.681	3.055		12
	13	1.350	1.771	2.160	2.650	3.012		13
	14	1.345	1.761	2.145	2.624	2.977		14
	15	1.341	1.753	2.131	2.602	2.947		15
	16	1.337	1.746	2.120	2.583	2.921		16
	17	1.333	1.740	2.110	2.567	2.898		17
	18	1.330	1.734	2.101	2.552	2.878		18
	19	1.328	1.729	2.093	2.539	2.861		19
	20	1.325	1.725	2.086	2.528	2.845		20
	21	1.323	1.721	2.080	2.518	2.831		21
	22	1.321	1.717	2.074	2.508	2.819		22
	23	1.319	1.714	2.069	2.500	2.807		23
	24	1.318	1.711	2.064	2.492	2.797		24
	25	1.316	1.708	2.060	2.485	2.787		25
	26	1.315	1.706	2.056	2.479	2.779		26
	27	1.314	1.703	2.052	2.473	2.771		27
	28	1.313	1.701	2.048	2.467	2.763		28
	29	1.311	1.699	2.045	2.462	2.756		29
	30	1.310	1.697	2.042	2.457	2.750		30
	31	1.309	1.694	2.037	2.449	2.738		32
	35	1.306	1.690	2.030	2.438	2.725		35
	40	1.303	1.684	2.021	2.423	2.704		40
	45	1.301	1.679	2.014	2.412	2.690		45
	50	1.299	1.676	2.009	2.403	2.678		50
	60	1.296	1.671	2.000	2.390	2.660		60
	70	1.294	1.667	1.994	2.381	2.648		70
	80	1.292	1.664	1.990	2.374	2.639		80
	100	1.290	1.660	1.984	2.364	2.626		100
	120	1.289	1.658	1.980	2.358	2.617		120
	140	1.288	1.656	1.977	2.353	2.611		140
	160	1.286	1.654	1.974	2.350	2.607		160
	180	1.286	1.653	1.973	2.347	2.603		180
	200	1.286	1.653	1.972	2.345	2.601		200
	250	1.285	1.651	1.969	2.341	2.596		250
	300	1.284	1.650	1.968	2.339	2.592		300
	400	1.284	1.649	1.966	2.336	2.588		400
	500	1.283	1.648	1.965	2.334	2.586		500
	∞	1.282	1.645	1.960	2.326	2.576		∞
Confidence levels		80%	90%	95%	98%	99%		

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Conditions

- Randomization condition.
- 10% condition.
- Nearly normal condition.

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Randomization Condition

- Data arise from a random sample from some population.
- Data arise from a randomized experiment.

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10% Condition

- The sample is no more than 10% of the population.
- Not as critical for means as it is for proportions.

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Nearly Normal Condition

- The data come from a population whose shape is unimodal and symmetric.
  - Look at the distribution of the sample.
  - Could the sample have come from a normal model?

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Confidence Interval for μ

$\bar{y} - t^*_{n-1}\mathrm{SE}(\bar{y})$ to $\bar{y} + t^*_{n-1}\mathrm{SE}(\bar{y})$

t^*_{n-1} is from Table T

$$\mathrm{SE}(\bar{y}) = \frac{s}{\sqrt{n}}$$

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Table T

df					
1					
2					
3					
4					
⋮					
$n-1$	$\rightarrow t^*_{n-1}$				
Confidence Levels	80%	90%	95%	98%	99%

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Inference for μ

- What is the mean heart rate for all young adults?
- Use the sample mean heart rate, $\bar{y}$, to make inferences about the population mean heart rate, μ.

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Sample Data

- Random sample of n = 25 young adults.
- Heart rate – beats per minute

  70, 74, 75, 78, 74, 64, 70, 78, 81, 73

  82, 75, 71, 79, 73, 79, 85, 79, 71, 65

  70, 69, 76, 77, 66

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Summary of Data

- $n = 25$
- $\bar{y} = 74.16$ beats
- $s = 5.375$ beats
- $\mathrm{SE}(\bar{y}) = \frac{s}{\sqrt{n}} = 1.075$ beats

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Conditions

- Randomization condition:
  - Met because we have a random sample of 25 young adults.
- 10% condition:
  - Met because 25 is less than 10% of all young adults that could have been sampled.

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Conditions

- Nearly Normal Condition
  - Could the sample have come from a population described by a normal model?

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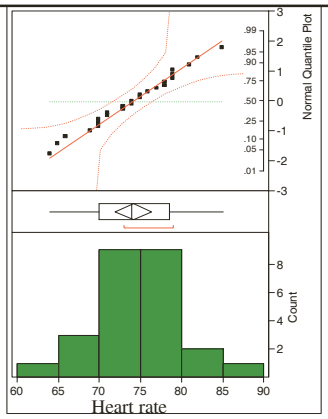

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Nearly Normal Condition

- Normal quantile plot – data follows straight line for a normal model.
- Box plot – symmetric.
- Histogram – unimodal and symmetric.

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Confidence Interval for μ

$\bar{y} - t^*_{n-1}\mathrm{SE}(\bar{y})$ to $\bar{y} + t^*_{n-1}\mathrm{SE}(\bar{y})$

t^*_{n-1} is from Table T

$$\mathrm{SE}(\bar{y}) = \frac{s}{\sqrt{n}}$$

25

Table T

df					
1					
2					
3					
4					
⋮					
24			**2.064**		
Confidence Levels	80%	90%	95%	98%	99%

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Confidence Interval for μ

$\bar{y} - t^*_{n-1}\mathrm{SE}(\bar{y})$ to $\bar{y} + t^*_{n-1}\mathrm{SE}(\bar{y})$

$74.16 \pm 2.064(1.075)$

$74.16 - 2.22$ to $74.16 + 2.22$

71.94 beats to 76.38 beats

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Interpretation

- We are 95% confident that the population mean heart rate of young adults is between
  71.94 beats/min and 76.38 beats/min

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Interpretation

- Plausible values for the population mean.
- 95% of intervals produced using random samples will contain the population mean.

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JMP: Analyze – Distribution

Mean	74.16
Std Dev	5.375
Std Err Mean	1.075
Upper 95% Mean	**76.38**
Lower 95% Mean	**71.94**
N	25

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