Example

\[ \hat{p} = 0.57 \]

\[ \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{\frac{0.57(0.43)}{1130}} = 0.015 \]

0.57 - 2(0.015) to 0.57 + 2(0.015)

0.54 to 0.60

Interpretation

* We are 95% confident that the population proportion of all adults in the U.S. who believe abortion should be legal is between 54% and 60%.

Interpretation

* Plausible values for the population parameter \( p \).
* 95% confidence in the process that produced this interval.
95% Confidence

If one were to repeatedly sample at random 1,130 adults and compute a 95% confidence interval for each sample, 95% of the intervals produced would contain, or capture, the population proportion \( p \).

Simulation

http://statweb.calpoly.edu/chance/applets/Confsim/Confsim.html
Margin of Error

\[ 2SE(\hat{p}) = 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]

Is called the Margin of Error (ME).
This is the furthest \( \hat{p} \) can be from \( p \), with 95% confidence.

---

What if we want to be 99.7% confident?

\[ ME = 3SE(\hat{p}) = 3\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]

---

Margin of Error

\[ ME = z^* \cdot SE(\hat{p}) = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]

<table>
<thead>
<tr>
<th>Confidence</th>
<th>80%</th>
<th>90%</th>
<th>95%</th>
<th>98%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z^* )</td>
<td>1.282</td>
<td>1.645</td>
<td>2 or 1.96</td>
<td>2.326</td>
<td>2.576</td>
</tr>
</tbody>
</table>
Another Example

* “Do you believe the theory that increased carbon dioxide and other gases released into the atmosphere will, if unchecked, lead to global warming and an increase in average temperatures?”

Another Example

n=1,052 randomly selected adults.

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
<th>Unsure</th>
</tr>
</thead>
<tbody>
<tr>
<td>71%</td>
<td>23%</td>
<td>6%</td>
</tr>
</tbody>
</table>

Another Example

* 90% confidence interval for $p$, the proportion of the population of all adults in the U.S. who believe the theory that increased carbon dioxide and other gases released into the atmosphere will, if unchecked, lead to global warming and an increase in average temperatures
Calculation

\[ \hat{p} = 0.71 \quad \text{SE}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.014 \quad z^* = 1.645 \]

\[ 0.71 - 1.645(0.014) \text{ to } 0.71 + 1.645(0.014) \]
\[ 0.71 - 0.023 \text{ to } 0.71 + 0.023 \]
\[ 0.687 \text{ to } 0.733 \]

What Sample Size?

**Conservative Formula**

- The sample size to be 95% confident that \( \hat{p} \), the sample proportion, will be within ME of the population proportion, \( p \).

\[ n = \frac{1}{ME^2} \]

Example

- Suppose we want to be 95% confident that our sample proportion will be within 0.02 of the population proportion.

\[ n = \frac{1}{ME^2} \Rightarrow n = \frac{1}{(0.02)^2} = 2,500 \]