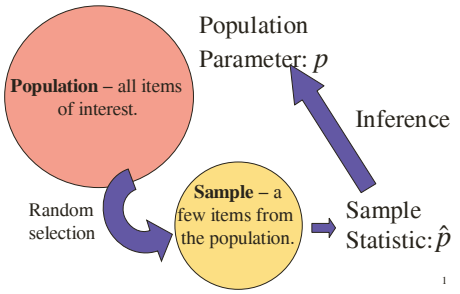
Stat 101L: Lecture 26

Sampling Distribution Models



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Sampling Distribution of $\hat{p}$

- Shape: Approximately Normal
- Center: The mean is p.
- Spread: The standard deviation is $\sqrt{\frac{p(1-p)}{n}}$

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Sampling Distribution of $\hat{p}$

- Conditions:
  - 10% Condition: The size of the sample should be less than 10% of the size of the population.
  - Success/Failure Condition: np and n(1 – p) should both be greater than 10.

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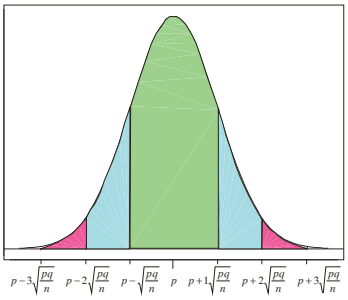
68 – 95 – 99.7 Rule



Probability

- If the population proportion, p, is known, we can find the probability or chance that $\hat{p}$ takes on certain values using a normal model.

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Inference

- In practice the population parameter, p, is not known and we would like to use a sample to tell us something about p.
- Use the sample proportion, $\hat{p}$, to make inferences about the population proportion p.

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Example

- Population: All adults in the U.S.
- Parameter: Proportion of all adults in the U.S. who feel that abortion should be legal. Unknown!

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Example

- Sample: 1,130 randomly selected adults nationwide. ABC News – Washington Post Poll, Jan. 9 – 12, 2008.
- Statistic: 644 of the 1,130 adults in the sample (57%) answered that abortion should be legal.

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68-95-99.7 Rule

- 95% of the time the sample proportion, $\hat{p}$, will be between

$$p - 2\sqrt{\frac{p(1-p)}{n}} \text{ and } p + 2\sqrt{\frac{p(1-p)}{n}}$$

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68-95-99.7 Rule

- 95% of the time the sample proportion, $\hat{p}$, will be within

$$2\sqrt{\frac{p(1-p)}{n}}$$

two standard deviations of p.

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Standard Deviation

- Because p, the population proportion is not known, the standard deviation

$$\mathrm{SD}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

is also unknown.

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Standard Error

- Substitute $\hat{p}$ as our estimate (best guess) of p.
- The standard error of $\hat{p}$ is:

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

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- About 95% of the time the sample proportion, $\hat{p}$, will be within

$$2SE(\hat{p}) = 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

two standard errors of p.

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- About 95% of the time the sample proportion, p, will be within

$$2SE(\hat{p}) = 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

two standard errors of $\hat{p}$.

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Confidence Interval for p

- We are 95% confident that p will fall between

$$\hat{p} - 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \text{ and } \hat{p} + 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

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