Probability

- Subjective (Personal)
  - Based on feeling or opinion.
- Empirical
  - Based on experience.
- Theoretical (Formal)
  - Based on assumptions.

The Deal

- Bag o' chips (poker chips).
  - Some are red.
  - Some are white.
  - Some are blue.
  - Draw a chip from the bag.

The Deal

- Draw a blue chip give 2 Extra Credit Points.
- Draw a red chip give 1 Extra Credit Points.
- Draw a white chip take 2 Extra Credit Point away.
Is this a good deal?

- Subjective (personal) probability
  - Based on your beliefs and opinion.
- Empirical probability
  - Based on experience.
  - Conduct a series of trials.
  - Each trial has an outcome (R, W, B).

Empirical Probability

- Look at the long run relative frequency of each of the outcomes.
  - Blue
  - Red
  - White

Theoretical Probability

- Look in the bag and see how many
  - Blue chips -
  - Red chips -
  - White chips -
- Assumption
  - Each chip has the same probability of being chosen. Equally likely.
Law of Large Numbers
- For repeated independent trials, the long run relative frequency of an outcome gets closer and closer to the true probability of the outcome.

Formal Probability
- A probability is a number between 0 and 1.
- Something has to happen rule.
  - The probability of the set of all possible outcomes of a trial must be 1.

Formal Probability
- Event - a collection of outcomes.
  - Win extra credit points (Blue or Red chip)
- Complement rule
  - The probability an event occurs is 1 minus the probability that it doesn’t occur.
  - \( P(A) = 1 - P(A^c) \)
Formal Probability

- Disjoint events – no outcomes in common.
- Addition Rule for disjoint events.
  - $P(A \text{ or } B) = P(A) + P(B)$
  - $P(\text{Blue or Red}) = P(\text{Blue}) + P(\text{Red})$

Formal Probability

- Independent trials
- Multiplication rule for independent trials.
  - $P(\text{outcome 1}^\text{st} \text{ and outcome 2}^\text{nd}) = P(\text{outcome 1}^\text{st}) \times P(\text{outcome 2}^\text{nd})$

Example

- What is the chance that two draws in a row will result in everyone getting extra credit points?

  
  $P(\text{win 1}^\text{st} \text{ and win 2}^\text{nd}) = P(\text{win 1}^\text{st}) \times P(\text{win 2}^\text{nd})$
  
  $P(\text{win 1}^\text{st}) = P(\text{Blue or Red}) = P(\text{Blue}) + P(\text{Red})$
  
  $P(\text{win 2}^\text{nd}) = P(\text{Blue or Red}) = P(\text{Blue}) + P(\text{Red})$