Stat 101L: Lecture 12

Algebra Review

- The equation of a straight line
- $y = mx + b$
  - m is the slope – the change in y over the change in x – or rise over run.
  - b is the y-intercept – the value where the line cuts the y axis.

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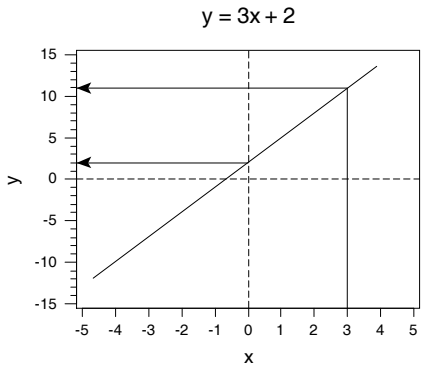

Review

- $y = 3x + 2$
  - $x = 0$ ⇒ $y = 2$ (y-intercept)
  - $x = 3$ ⇒ $y = 11$
  - Change in y (+9) divided by the change in x (+3) gives the slope, 3.

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Linear Regression

- Example: Tar (mg) and nicotine (mg) in cigarettes.
  - y, Response: Nicotine (mg).
  - x, Explanatory: Tar (mg).
  - Cases: 25 brands of cigarettes.

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Correlation Coefficient

- Tar and nicotine

$$r = \frac{\sum z_x z_y}{n-1} = \frac{22.9437}{24}$$

- $r = 0.956$

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Linear Regression

- There is a strong positive linear association between tar and nicotine.
- What is the equation of the line that models the relationship between tar and nicotine?

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Linear Model

- The linear model is the equation of a straight line through the data.
- A point on the straight line through the data gives a predicted value of y, denoted $\hat{y}$.

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Residual

- The difference between the observed value of y and the predicted value of y, $\hat{y}$, is called the residual.
- Residual = $y - \hat{y}$

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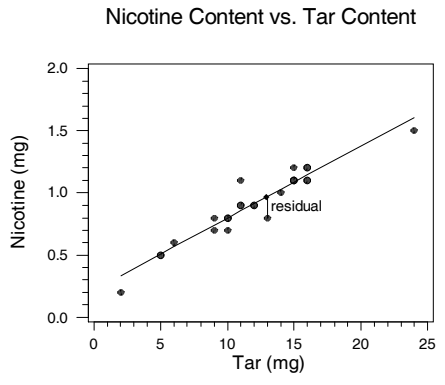

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Line of "Best Fit"

- There are lots of straight lines that go through the data.
- The line of "best fit" is the line for which the sum of squared residuals is the smallest – the least squares line.

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Line of "Best Fit"

$$\hat{y} = b_0 + b_1 x$$

Least squares slope: $b_1 = r\frac{s_y}{s_x}$

intercept: $b_0 = \bar{y} - b_1\bar{x}$

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Summary of the Data

Tar, x	Nicotine, y
$\bar{x} = 11.92\,\text{mg}$	$\bar{y} = 0.908\,\text{mg}$
$s_x = 4.636\,\text{mg}$	$s_y = 0.2812\,\text{mg}$
$r = 0.956$	

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Least Squares Estimates

$$b_1 = 0.956\frac{0.2812}{4.636} = 0.058$$

$$b_0 = 0.908 - 0.058(11.92) = 0.217$$

$$\hat{y} = 0.217 + 0.058x$$

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Interpretations

- Slope – for every 1 mg increase in tar, the nicotine content increases, on average, 0.058 mg.
- Intercept – there is not a reasonable interpretation of the intercept in this context because one wouldn't see a cigarette with 0 mg of tar.

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