Statistics 101 – Homework 7

Solution

Assignment:

1. Do the following problems from the text, *Intro Stats*, 3rd Edition. If you have an earlier edition of the text, check with someone who has the 3rd Edition to make sure you do the correct problems.

   a) Chapter 14 – problems 31 and 32.
   b) Chapter 18 – problems 3, 4, 11, 13 and 14.

   If you have questions about the problems in the book, see your course instructor.

2. The maker of M&M's says on its website that 16% of Dark Chocolate M&M's are orange. Suppose that M&M's are packaged at random. We wish to examine the sample proportion of orange Dark Chocolate M&M's, $\hat{p}$, in various sized bags.

   a) For each of the different sized bags, give the mean and standard deviation of the sampling distribution of $\hat{p}$. Also comment on whether or not the success/failure condition is met for the sampling distribution to be approximately normal.

      i) Fun size bags containing 25 Dark Chocolate M&M's.

      Sampling distribution of $\hat{p}$:

      mean = 0.16, standard deviation = $\sqrt{\dfrac{0.16(0.84)}{25}} = 0.073$

      Success/Failure condition is not met. np = 4 which is less than 10.

      ii) Small bags containing 64 Dark Chocolate M&M's.

      Sampling distribution of $\hat{p}$:

      mean = 0.16, standard deviation = $\sqrt{\dfrac{0.16(0.84)}{64}} = 0.046$

      Success/Failure condition is met. np = 10.24 and n(1–p) = 53.76 which are both at least 10.

      iii) Medium bags containing 100 Dark Chocolate M&M's.

      Sampling distribution of $\hat{p}$:

      mean = 0.16, standard deviation = $\sqrt{\dfrac{0.16(0.84)}{100}} = 0.037$

      Success/Failure condition is met. np = 16 and n(1-p)=84 which are both at least 10.

iv) Extra large bags containing 400 Dark Chocolate M&M's

Sampling distribution of $\hat{p}$:

mean = 0.16, standard deviation = $\sqrt{\frac{0.16(0.84)}{400}} = 0.018$

Success/Failure condition is met. np = 64 and n(1–p)=336 which are both at least 10.

b) For the extra large bags containing 400 Dark Chocolate M&M's, use the 68-95-99.7 Rule to describe how the sample proportion of orange Dark Chocolate M&M's might vary from bag to bag.

68% of the time the sample proportion of yellow will fall between
0.16 – 0.018 = 0.142 and 0.16 + 0.018 = 0.178
95% of the time the sample proportion of yellow will fall between
0.16 – 2(0.018) = 0.124 and 0.16 + 2(0.018) = 0.196
99.7% of the time the sample proportion of yellow will fall between
0.16 – 3(0.018) = 0.106 and 0.16 + 3(0.018) = 0.214

c) In an extra large bag of 400 Dark Chocolate M&M's there are 80 orange. Is this an unusually large number of orange? Explain.

$$\hat{p} = \frac{80}{400} = 0.20 \quad z = \frac{0.20 - 0.16}{0.018} = 2.22 \quad \text{Table Z} : 0.9868$$
$$1 - 0.9868 = 0.0132$$

The chance of getting 80 or more orange Dark Chocolate M&M's in a random sample of 400 is only 1.3%. This is unlikely therefore 80 or more (out of 500) orange M&M's is an unusually large number.

3. It is believed that 44% of all college students in the United States engage in binge drinking (5 or more drinks at a sitting for men, 4 or more for women). Consider a random sample of 100 college students. Verify that the success/failure condition is met. Use the 68-95-99.7 Rule to describe the sampling distribution model for the sample proportion of students who engage in binge drinking.

Success/Failure condition: Both np = 44 and n(1-p) = 56 are at least 10. This verifies that this condition is met.

Sampling distribution of $\hat{p}$:

mean = 0.44, standard deviation = $\sqrt{\frac{0.44(0.56)}{100}} = 0.05$

68% of the time the sample proportion of binge drinkers will fall between
0.44 – 0.05 = 0.39 and 0.44 + 0.05 = 0.49
95% of the time the sample proportion of binge drinkers will fall between
0.44 – 2(0.05) = 0.34 and 0.44 + 2(0.05) = 0.54
99.7% of the time the sample proportion of binge drinkers will fall between
0.44 – 3(0.05) = 0.29 and 0.44 + 3(0.05) = 0.59

4. In 2004, 20.9% of all adults (18 years old or older) in the United States were current smokers. There were approximately 220,000,000 adults in the United States in 2004. For a random sample of 1000 U.S. adults is the 10% condition met? Explain briefly. Is the success/failure condition met? Explain briefly. Use the 68-95-99.7 Rule to describe the sampling distribution model for the proportion current smokers in a random sample of 1000 adults in the United States in 2004.

10% condition is met. 1000 is much less than 10% of 220,000,000.

Success/Failure condition is met. np = 209 and n(1-p) = 791 both of which are at least 10.

Sampling distribution of $\hat{p}$ **:**

mean = 0.209, standard deviation = $\sqrt{\frac{0.209(0.791)}{1000}} = 0.013$

68% of the time the sample proportion of smokers will fall between
0.209 – 0.013 = 0.196 and 0.209 + 0.013 = 0.222
95% of the time the sample proportion of smokers will fall between
0.209 – 2(0.013) = 0.183 and 0.209 + 2(0.013) = 0.235
99.7% of the time the sample proportion of smokers will fall between
0.209 – 3(0.013) = 0.170 and 0.209 + 3(0.013) = 0.248

5. A company who packages microwave popcorn claims that only 5% of the popcorn in each package fails to pop. A package of popcorn contains 200 randomly selected kernels of popcorn. What is the chance that a package will have 16 or more kernels that fail to pop? Verify that the appropriate conditions for computing this probability are met?

Sampling distribution of $\hat{p}$ **:**

mean = 0.05, standard deviation = $\sqrt{\frac{\mathbf{0.05(0.95)}}{\mathbf{200}}} = \mathbf{0.0154}$

$$\hat{p} = \frac{16}{200} = 0.08, \quad z = \frac{0.08 - 0.05}{0.0154} = 1.95$$

Table Z : 0.9738

$1 - 0.9738 = 0.0272$ **or about a 2.7% chance**

10% condition is met. 200 kernels is much less than 10% of all the popcorn kernels from the company.

Success/Failure condition is met. np = 10 and n(1-p) = 190 both of which are at least 10.