Aerial photographs and visual sightings are often used to monitor the number and location of wildlife. Often measurements of physical characteristics are necessary to monitor the health of animals. The length of alligators can be estimated quite well from visual sightings or photographs. Weight, however, is a different matter. Below are data on the length (in meters) and weight (in kilograms) for alligators captured in Florida.

<table>
<thead>
<tr>
<th>Alligator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, x</td>
<td>2.4</td>
<td>1.9</td>
<td>1.5</td>
<td>2.2</td>
<td>2.4</td>
<td>1.6</td>
<td>2.2</td>
<td>1.8</td>
</tr>
<tr>
<td>Weight, y</td>
<td>58</td>
<td>23</td>
<td>13</td>
<td>36</td>
<td>50</td>
<td>15</td>
<td>41</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length, x</th>
<th>Weight, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.0 m</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.3505 m</td>
</tr>
</tbody>
</table>

Correlations

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>1.0000</td>
<td>0.9595</td>
</tr>
<tr>
<td>Weight</td>
<td>0.9595</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

- Compute the least squares slope, \( b_1 \).

\[
b_1 = r \frac{s_y}{s_x} = 0.9595 \frac{17.246}{0.3505} = 47.21 \text{ kg/m}
\]

- Give an interpretation of the slope within the context of the problem.

**For each additional 1 meter of length, the weight of an alligator increases by 47.21 kg, on average.**
• Compute the least squares y-intercept, $b_0$.

$$b_0 = \bar{y} - b_1 \bar{x} = 31.5 - 47.21(2) = 31.5 - 94.42 = -62.92 \text{ kg}$$

• Give the equation of the least squares regression line and use this line to predict the weight of an alligator with a length of 2.5 meters.

$$\hat{y} = -62.92 + 47.21x$$

$$\hat{y} = -62.92 + 47.21(2.5) = -62.92 + 118.025 = 55.105 \text{ kg}$$

• Plot the least squares regression line on the graph on the first page. It must be clear to me that you used the equation of the line to plot the line.

$$x = 2.5, \hat{y} = 55.105$$

$$x = 1.5, \hat{y} = 7.895$$

• Below is a plot of residuals. What does this plot tell about the appropriateness of the linear model relating length to weight?

For short and long alligators, the line tends to under-predict the weight. For alligators of medium length, the line tends to over-predict. This curved pattern indicates that a curved relationship would do even better than the straight line model.

Formulas

$$\bar{y} = \frac{\sum y}{n} \quad s_y = \sqrt{\frac{\sum (y - \bar{y})^2}{n-1}} \quad \bar{x} = \frac{\sum x}{n} \quad s_x = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$r = \frac{\sum z_x z_y}{n-1} \quad z_x = \frac{x - \bar{x}}{s_x} \quad z_y = \frac{y - \bar{y}}{s_y} \quad r = \frac{\sum (x - \bar{x})(y - \bar{y})}{(n-1)s_x s_y}$$

$$b_1 = \frac{r s_y}{s_x} \quad b_0 = \bar{y} - b_1 \bar{x} \quad \hat{y} = b_0 + b_1 x$$