Inference

- Confidence Interval for \( p \)

\[
\hat{p} - z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \text{to} \quad \hat{p} + z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]

Confidence Interval

- Plausible values for the unknown population proportion, \( p \).
- We have confidence in the process that produced this interval.

Confidence Interval

- The population proportion, \( p \), could be any of the values in the interval.
- Values outside the interval are not plausible values for \( p \).
Inference: Hypothesis Test

- Propose a value for the population proportion, $p$.
- Does the sample data support this value?

Example

- A law firm will represent people in a class action lawsuit against a car manufacturer only if it is sure that more than 10% of the cars have a particular defect.

Example

- Population: All cars of a particular make and model.
- Parameter: Proportion of all the cars of this make and model that have a particular defect, $p$. 
Example

- Null Hypothesis
  - $H_0: p = 0.10$
- Alternative Hypothesis
  - $H_A: p > 0.10$

Example

- The law firm randomly selects 100 people who own the particular make and model of the car and finds out that 12 of them have cars that have the defect.
- Is this sufficient evidence for the law firm to proceed with the class action law suit?

Example

- How likely is it to get a sample proportion as extreme as the one we observe when taking a random sample of 100 from a population with $p = 0.10$?
Example

- Sampling distribution of $\hat{p}$
  - Shape approximately normal because 10% condition and success/failure condition satisfied.
  - Mean: $p = 0.10$
  - Standard Deviation: $\sqrt{\frac{0.10(0.90)}{100}} = 0.03$

Draw a Picture

Standardize

$$ z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} $$

$$ z = \frac{0.12 - 0.10}{\sqrt{\frac{0.10(0.90)}{0.03}}} = \frac{0.02}{0.03} = 0.67 $$