• About 95% of the time the sample proportion, \( \hat{p} \), will be within

\[
2SE(\hat{p}) = 2 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]

two standard errors of \( p \).

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\]

two standard errors of \( \hat{p} \).

Confidence Interval for \( p \)
• We are 95% confident that \( p \) will fall between

\[
\hat{p} - 2 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \text{and} \quad \hat{p} + 2 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]
Example

\( \hat{p} = 0.40 \)

\[ \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.40(0.60)}{1004}} = 0.0155 \]

0.40 – 2(0.0155) to 0.40 + 2(0.0155)

0.369 to 0.431

Interpretation

• We are 95% confident that the population proportion of all adults in the U.S. who believe climate change is a real threat is between 36.9% and 43.1%.

Interpretation

• Plausible values for the population parameter \( p \).
• 95% confidence in the process that produced this interval.
95% Confidence

• If one were to repeatedly sample at random 1004 adults in the U.S. and compute a 95% confidence interval for each sample, 95% of the intervals produced would contain the population proportion $p$.

Simulation

http://statweb.calpoly.edu/chance/applets/Confsim/Confsim.html
Margin of Error

\[ 2SE(\hat{p}) = 2\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \]

Is called the Margin of Error (ME).
This is the furthest \( \hat{p} \) can be from \( p \), with 95% confidence.

Margin of Error

- What if we want to be 99.7% confident?

\[ ME = 3SE(\hat{p}) = 3\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \]

Margin of Error

\[ ME = z^* SE(\hat{p}) = z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \]

<table>
<thead>
<tr>
<th>Confidence</th>
<th>80%</th>
<th>90%</th>
<th>95%</th>
<th>98%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z^* )</td>
<td>1.282</td>
<td>1.645</td>
<td>2 or 1.96</td>
<td>2.326</td>
<td>2.576</td>
</tr>
</tbody>
</table>