

# The Effect of Dropping a Ball from Different Heights on the Number of Times the Ball Bounces

Or: How I Learned to Stop Worrying and Love the Ball

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January 29, 1964

**Comment [DP1]:** Titles, headings, and figure/table captions offer special opportunities for engaging your readers, identifying the scope of your report, establishing the report's logic, or highlighting a main point. On first reading, these elements invite your audience to share in your thinking; on subsequent readings, they help readers find information or recall key points.

### *The importance of proving intuition with experimentation*

When it comes to the basic physical laws of the universe, our intuition often plays tricks on us. For example, we may think that a bowling ball will fall faster than a marble, but gravity's hold on the lighter marble is just as strong as its hold on the heavier bowling ball, and the two travel to the ground at exactly the same speed. Time and again, what we think should happen is proven wrong by what actually happens. This is why we need to rigorously test all of our assumptions about the universe.

In the following report, we rigorously test the assumption that a ball dropped from a higher height will bounce more times than a ball dropped from a lower height. We believe this assumption because our intuition tells us that the higher a ball is before it is dropped, the more speed it will pick up as it is falling. A ball hitting the ground with greater speed should bounce higher than a ball striking the ground with less speed; therefore, a ball dropped from a greater height will bounce higher than a ball dropped from a lesser height. When a ball is bouncing high into the air, it will continue to bounce for some time. By contrast, a low-bouncing ball will quickly run out of room between the height of each bounce and the ground, and, as such, will soon stop bouncing altogether. Therefore, it only makes sense that a ball will bounce more often when it is released from a greater height. But to prove our conjecture about the physics behind bouncing balls, we cannot rely on our intuition. We need to use a well-designed experiment to investigate the relationship between the height of a falling ball's release point and how many times the ball bounces before coming to rest.

### *Testing our assumptions about the physics of bouncing balls*

In our experiment we dropped thirty Wilson brand Extra Duty tennis balls from five different heights. We used tennis balls in our study because, unlike baseballs or billiards for example, tennis balls are designed to bounce when dropped. We could have used any type of bouncing balls in our experiment—racquetballs, basketballs, etc.—what is important is that all thirty balls used in our study were the same type of ball. By exclusively using tennis balls rather than using a mixture of different types of balls we controlled the intrinsic "bounciness" of the balls used in the experiment. We further controlled the intrinsic bounciness of the balls by ensuring that all thirty tennis balls used in the study were made by the same manufacturer (Wilson), were the same model (Extra Duty) and—because old tennis balls are notorious for losing their bounce as they age—using only brand new tennis balls.

Each tennis ball used in the experiment was randomly assigned to be dropped from one of five heights: 1 meter, 1.5 meters, 2 meters, 2.5 meters or 3 meters. An equal number of tennis balls were assigned to each height so by the end of the experiment we had dropped six tennis balls from the 1

**Comment [A2]:** A descriptive heading helps readers understand one or more key aspects of a section: its main topic, its main purpose, its main conclusion, its benefit for readers.

**Comment [DP3]:** This paragraph clearly states the logic behind the conjecture, in effect inviting readers to agree or disagree and thus to read on to see if their thinking is confirmed by the experiment.

**Comment [A4]:** The conjecture about the answer to the research question

**Comment [A5]:** The research question

**Comment [A6]:** The explanatory variable is the height of the ball's release point, and the response variable is the number of times the ball bounces before coming to rest.

**Comment [A7]:** The experimental units

**Comment [A8]:** A control in the experiment

**Comment [A9]:** Another control in the experiment

**Comment [A10]:** Three more controls in the experiment. Because you are likely to have numerous controls, think carefully about how to introduce and organize them. Here, for example, several controls are organized under the concept of "bounciness." Sometimes categorizing types of controls can help readers remember the key control issues without getting lost in long list of controls.

**Comment [A11]:** An example of randomization

**Comment [A12]:** Five levels of the experimental factor

meter height, six tennis balls from 1.5 meters, etc. In order to guarantee that we did not apply any extra downward thrust when dropping the tennis balls we dropped each tennis ball by placing it into a box equipped with a double-flapped trapdoor. We held the bottom of the box to the desired drop height and released the trapdoor, allowing the tennis ball to fall to the ground under just the force of gravity. Not only did this method of dropping the tennis balls ensure that each tennis ball was released without the push of a human hand, it also standardized the method of measuring the release point. It is much easier to consistently measure to the bottom of a flat box than it would be to measure to the bottom tip of a round tennis ball. A picture of the box and trapdoor used in this experiment is displayed in Figure 1 below.

Figure 1: Box Used to Drop Tennis Ball



The tennis balls were all dropped onto a smooth and level concrete floor inside a room where a constant temperature of 72°F was maintained and no breeze was observed.

Two members of the experiment team were assigned to count how many times each tennis ball bounced. We used two counters instead of just one because it is sometimes difficult to get an accurate tally of how many times a ball bounces, especially when the ball is rebounding low to the ground right before it is about to stop bouncing. Often the two counters counted the same number of bounces, but whenever the two counters disagreed on the number of times a ball bounced we recorded the average number of bounces tallied by each counter. This sometimes resulted in the official bounce count being a fractional value. For example, the number of bounces recorded for one of the tennis balls dropped from 2 meters is 1.5. Obviously a tennis ball cannot bounce half a time. The 11.5 bounces was recorded for this tennis ball because one counter believed that the ball bounced 12 times while the other counter

**Comment [A13]:** Replication. To discover any missing details or unclear instructions, have someone read this experimental design section of your report and explain how they would replicate your study.

**Comment [A14]:** Another control

**Comment [A15]:** It may be helpful to include pictures of your experimental setup with explanatory captions.

**Comment [A16]:** Another control

**Comment [A17]:** Another control

**Comment [A18]:** To replicate or modify your study, readers need to know how you measured your data.

only counted 11 bounces. To keep the counting as consistent as possible, the same two members of the experiment team acted as counters for the duration of the experiment.

**Comment [A19]:** Another control

Because the counters could have become better at counting the number of bounces, we decided to randomize the order of the trials. Instead of dropping all of the tennis balls assigned to the 1 meter release point first and then dropping all of the balls assigned to the 1.5-meter release point, etc., we mixed up the order in which the balls were dropped. So, the first ball dropped was actually from the 2-meter release point, followed by two balls dropped from the 3-meter release point and then one ball from the 1.5-meter release point, etc. By randomizing the order of the trials, we avoided the potential problem of having the least accurate counts for the 1-meter release point and the most accurate counts for the 3-meter release point.

**Comment [A20]:** Randomization: this time, in the order of the trials instead of the assignment of experimental units

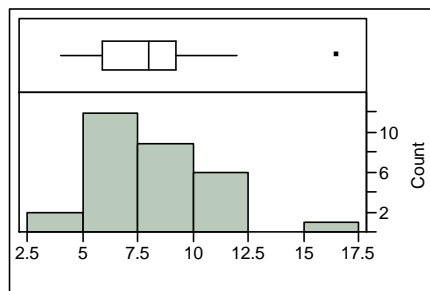
#### Modeling Upside-down-V Shaped Data

The first look at our data examines the distribution of our response variable. A histogram and boxplot of the response variable are shown in Figure 2. The distribution of the response variable is unimodal with an outlier at 16.5 bounces. Excluding the outlier, the distribution is slightly skewed right. The median number of bounces is 8, with an interquartile range of 3, and a range of 12.5.

**Comment [A21]:** Describe the shape of the distribution of the response variable.

**Comment [A22]:** Describe the center and spread of the response variable.

Figure 2: Distribution of Number of Bounces



Both variables collected in this experiment are summarized in Figure 3, a scatterplot of the height of each tennis ball's release point and the corresponding number of times each tennis ball bounced after it was dropped. We expect to see a strong, positive, linear relationship between the two variables, but, while the variables are strongly associated, it is clear from Figure 3 that the relationship is more complex than we anticipated. As the release height increases from 1 meter to 2 meters, the

**Comment [A23]:** Summarize your response variable using a histogram or stem-and-leaf display, noting its center and spread.

**Comment [A24]:** Notice that every figure in the report is discussed in the text.

**Comment [A25]:** A scatterplot is an important summary of two quantitative variables.

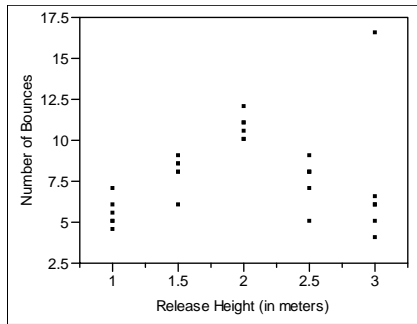
**Comment [A26]:** Relate your research conjecture to what you expect to see in the data

**Comment [A27]:** Did you see what you expected?

number of bounces increases linearly, as we expect, but when the release height increases from 2 meters to 3 meters the number of bounces—with the exception of one outlier at 3 meters and 16.5 bounces—actually decreases. This pattern in the data is best described as an upside-down letter “V.”

**Comment [A28]:** Describe the data. Is it linear? Strong or weak relationship? Outliers? Help readers understand patterns in the experimental data, especially any deviation from your conjecture. What are the implications of these patterns or deviations?

Figure 3: Tennis Balls' Release Heights vs. How Many Times the Ball Bounces



It is obvious that a single, straight line is not the best model for the upside-down-V shaped data observed in our experiment. The correlation between the release heights and the number of bounces is only 0.168, indicating that any linear relationship between the two variables is very weak. Furthermore, Figures 4 and 5 show us that when we force a linear model onto our non-linear data, the resulting regression line does an extremely poor job of approximating the actual data. The linear model consistently overestimates the number of bounces at the 1 meter and 3 meter release heights while consistently underestimating the number of bounces at the 2 meter release height.

**Comment [A29]:** Describe the correlation as another way to summarize the relationship and help readers interpret the data.

**Comment [A30]:** Describe what you see in the residual plot. How well does the linear model fit your data?

Figure 4: A Simple Linear Model Fit to All of the Experimental Data

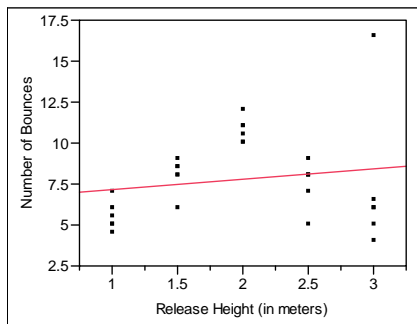
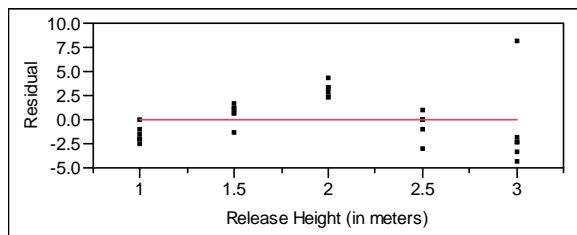


Figure 5: Residual Plot of the Linear Model Fit in Figure 4



While it is impossible to model our data with a single regression line, the upside-down-V shaped relationship observed in our experiment can be modeled with two regression lines. If we split the data in half and only consider the balls dropped from heights between 1 meter and 2 meters, then the relationship between a ball's release point and how many times it bounces can be described as strong, positive, linear and without outliers. For this type of data a linear model is appropriate and, as one can see in Figure 6, when we fit a linear model to this subset of the tennis balls, the regression line does an excellent job of approximating the observed data. Similarly, when we consider the balls dropped from heights ranging from 2 meters to 3 meters, the relationship between the balls' release points and the number of bounces is once again strong and linear. The only differences are that for this half of the data the relationship between the two variables is negative instead of positive and that there is an outlier amongst the higher release points while there are no outliers amongst the lower release points. Despite the outlier, a linear model fit to this second subset of tennis balls does a fair job of approximating the observed data, as one can see in Figure 7, and if we choose to remove the outlier from the data set, like in Figure 8, a linear model does an excellent job of approximating the observed data. So, while we cannot model all of the data with one regression line, we can split the data in half and use a different regression line for each half of the data. If we consider these two regression lines one model—albeit a more complex model than a simple linear model—we can successfully model all of the data in the experiment.

**Comment [A31]:** If the linear relationship is not good for your data, can you use two lines?

**Comment [A32]:** Now the linear model fits well, as illustrated in the figure.

**Comment [A33]:** Compare the differences between the two halves of the data.

**Comment [A34]:** If there is an outlier, you should fit two regression lines.

Figure 6: A Simple Linear Model Fit to Half of the Experimental Data (1 Meter to 2 Meters)

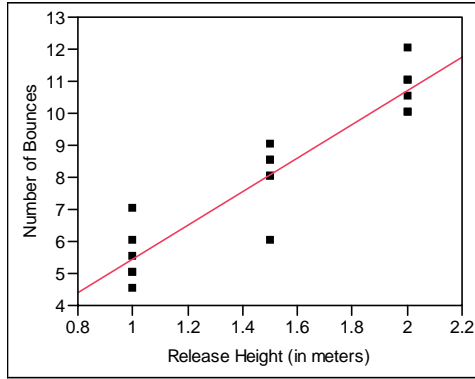


Figure 7: A Simple Linear Model Fit to Half of the Experimental Data (2 Meters to 3 Meters)

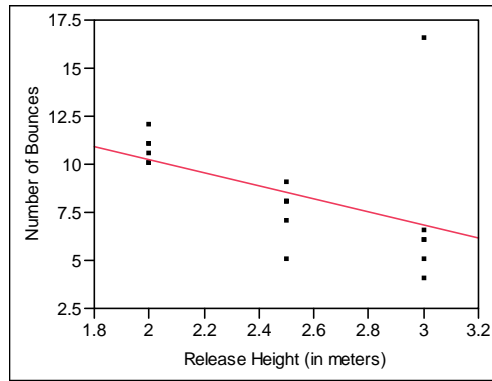
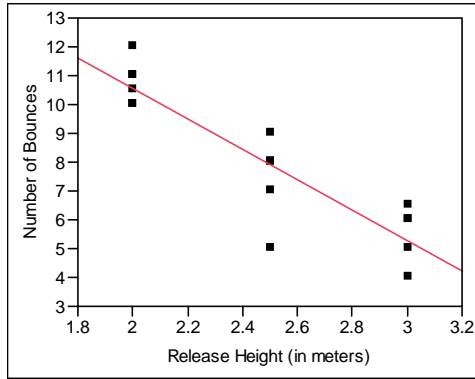


Figure 8: A Simple Linear Model Fit to Half of the Experimental Data (2 Meters to 3 Meters) Minus the Outlier



The equation of the regression line for tennis balls released from heights of 1 meter to 2 meters, i.e., the regression line displayed in Figure 6, is

**Comment [A35]:** Include the equation of the regression line.

$$\text{Predicted Number of Bounces} = 0.208 + 5.25 \cdot \text{Release Height} .$$

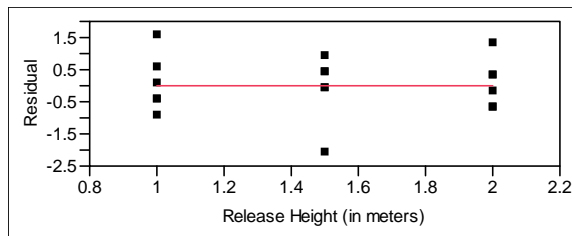
This means that for all tennis balls with release heights between 1 meter and 2 meters, a 1-meter increase in release height will increase the predicted number of times a ball bounces by 5.25. The R-squared value for this regression line is a very high 0.869, indicating that the regression line is a good fit for this half of the data. Also implying that our data is modeled well by this regression line is the residual plot for the model, displayed in Figure 9, which shows no pattern in the residuals.

**Comment [A36]:** Interpret the equation of the regression line.

**Comment [A37]:** Report and interpret R-squared

**Comment [A38]:** Look for patterns in the residual plot and interpret what you see.

Figure 9: Residual Plot for the Linear Model Fit in Figure 6

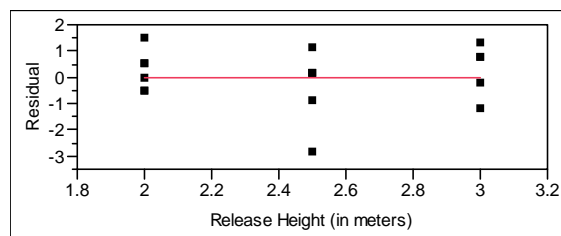


Deciding which of the two regression lines to use for the second half of the data—either the regression line plotted in Figure 7 or the line plotted in Figure 8—boils down to deciding whether or not the outlier at 16.5 bounces and the 3-meter release point should be included in the data analysis or if there is a logical reason for excluding the outlier. Because the tennis ball that produced this outlier hit the side of a wall before it fell to the floor, we choose to exclude this data point from the rest of our analysis. Therefore, the regression line for tennis balls dropped from 2 meters to 3 meters is the line plotted in Figure 8:

$$\text{Predicted Number of Bounces} = 21.129 - 5.29 \cdot \text{Release Height}.$$

Thus, for all tennis balls released at a height somewhere between 2 meters and 3 meters there is a decrease of 5.29 in the predicted number of bounces for every increase of 1 meter in release height. The high R-squared value of 0.811 and the random scatter of points in the residual plot displayed in Figure 10 tell us that this regression line is a very good model for this half of the data.

Figure 10: Residual Plot for the Linear Model Fit in Figure 8



The fact that the two regression lines are each excellent models for their respective halves of the data indicates that the overall model does a good job of representing all of the data from the experiment (excluding the outlier).

#### What Does It All Mean?

Once again, our intuition regarding the laws of the physical universe has been proven wrong by the laws of the physical universe. We hypothesized that a ball would bounce more frequently when released from a greater height, and while our experiment showed this to be true for drops of 1 to 2

meters, dropping a ball from any height greater than two meters actually reduces the number of times the ball bounces.

It is difficult to imagine any logical reason why this might be the case. One possibility is that the tennis balls that are released higher above the ground are, as we expected, hitting the ground with more force than the tennis balls released closer to the ground. Instead of using this extra force to bounce higher the balls dropped from 2 meters or more may be hitting the ground so hard that it actually damages the tennis balls, reducing the number of times the balls bounce. Admittedly this is explanation is fairly farfetched—the tennis balls used in this experiment are designed to hit the court travelling at 140 mph so it is unlikely that a freefall of a 3 meters would cause any significant damage.

Yet this explanation is worthy of further study. One could easily find a more durable tennis ball, repeat the experiment and see if this makes any difference in the results. We might also consider repeating the experiment with a wider range of release points, using another type of bouncing ball and dropping the balls onto a surface other than cement. Repeating the experiment with a wider range of release points would tell us if the two linear models that fit the data so well from 1 to 2 meters and 2 to 3 meters would continue to fit data for shorter and taller release points, respectively. If we replaced the tennis balls in the experiment with another type of bouncing ball, like racquetballs or basketballs for example, we could learn whether or not the unusual upside-down-V shaped data observed in this study is a result of some unique property of tennis balls or if it is indicative of a broader physical law. Repeating the original experiment (with tennis balls) on more traditional tennis surfaces such as grass or clay could yield more practical results for sports enthusiasts.

Finally, if this or a similar experiment were ever conducted again, the researchers might consider releasing the tennis balls (or whatever type of ball that is being used) in the center of a room, far away from all walls and furniture. We experienced some problems with tennis balls hitting a wall before they stopped bouncing and believe that this is the reason why there was an outlier in our data set at the 3-meter release point and 16.5 bounces.

**Comment [A39]:** Did you see what you guessed you would see? Recall the conjecture in the first section and your analysis from the fourth section. Provide your readers with a concise comparison of actual findings with anticipated findings.

**Comment [A40]:** Any speculation on why you didn't see what you expected? Now that readers understand your findings, help them rethink both the logic and the procedures behind your experimental design.

**Comment [A41]:** What would the next set of experiments look like?

**Comment [A42]:** Do the results generalize?

**Comment [A43]:** How could you improve the conduct of the experiment? Think of this concluding section as preparing your readers to replicate, redesign, or complement your study.

Data Appendix

Comment [A44]: Include your data as an appendix to the report.

Release Height (in meters)	Number of Bounces
1	5
1	5
1	4.5
1	7
1	6
1	5.5
1.5	8.5
1.5	6
1.5	9
1.5	8
1.5	8
1.5	8.5
2	10
2	10.5
2	12
2	11
2	10
2	11
2.5	8
2.5	8
2.5	7
2.5	8
2.5	9
2.5	5
3	6.5
3	6
3	6
3	4
3	5
3	16.5