Chapter 20
Planning Accelerated Life Tests

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Based on the authors’ text
Statistical Methods for Reliability

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8h 10min

Chapter 20 Objectives
Planning Accelerated Life Tests

20 - 1

Chapter 20 Objectives
Planning Accelerated Life Tests

• Outline reasons and practical issues in planning ALTs.
• Describe criteria for ALT planning.
• Illustrate how to evaluate the properties of ALTs.

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Possible Reasons for Conducting an Accelerated Test

Accelerated tests (ATs) are used for different purposes. These include:
• ATs designed to identify failure modes and other weaknesses in product design.
• ATs for improving reliability
• ATs to assess the durability of materials and components.
• ATs to monitor and audit a production process to identify changes in design or process that might have a seriously negative effect on product reliability.

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Motivation/Example
Reliability Assessment of an Adhesive Bond

• Need:
  • Estimate of the B10 of failure-time distribution at 50°C (expect ≥10 years).
  • Constraints
    • 300 test units.
    • 6 months for testing.
  • 50°C test expected to yield little relevant data.

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Model and Assumptions

• Failure-time distribution is loglocation-scale
  \[ \Pr(T \leq t) = F(t; \mu, \sigma) = \Phi\left[\log(t) - \frac{\mu}{\sigma}\right] \]

• \[ \mu = \mu(x) = \beta_0 + \beta_1 x, \]
  where \( x = \frac{\text{temp} + 273}{15} \).

• \[ \sigma \] does not depend on the experimental variables.

• Observations statistically independent.

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Assumed Planning Information for the Adhesive Bond Experiment

The objective is finding a test plan to estimate B10 with good precision.

• Weibull failure-time distribution with same shape parameter at each level of temperature and location scale parameter
  \[ \mu(x) = \beta_0 + \beta_1 x, \]
  where \( x \) is in the Arrhenius scale.

• 90% failing in 6 months at 120°C.
• 1% failing in 6 months at 50°C.

Result:
Defines failure probability in 6 months at all levels of temperature. If \( \sigma \) is given also, defines all model parameters.

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Engineers' Originally Proposed Test Plan for the Adhesive Bond

Temperature Allocation Failure Expected

<table>
<thead>
<tr>
<th>π</th>
<th>n</th>
<th>E(r)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>1/3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>1/3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1/3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Critique of Engineers' Original Proposed Plan

- Arrhenius model in doubt at high temperatures (above 120°C).
- Question ability to extrapolate to 50°C.
- Data much above the B10 are of limited value.

Suggestion for Improvement:

- Test at lower more realistic temperatures (even if only small fraction will fail).
- Larger allocation to lower temperatures.
- Artifactual model in doubt at high temperatures (above 120°C).
- Artifactual model in doubt at high temperatures (above 120°C).

Methods of Evaluating Test Plan Properties

Assume inferences needed on a function \( g(\theta) \) (one-to-one and all the first derivatives with respect to the elements of \( \theta \) exist and are continuous).

- Properties depend on test plan, model and (unknown) parameter values.
- Asymptotic variance of \( \theta \) needs a function \( \theta \) (one-to-one).

Simulation of Engineers' Modified Traditional ALT Plan

For this plan and the Weibull-Arrhenius model, \( A_{\text{log}(t_{50})} = 4.4167 \). The asymptotic precision factor for a 95% confidence interval of \( t_{50} \) is \( R = \exp(1.96 \times A_{\text{log}(t_{50})}) = 2.2622 \).
Statistically Optimum Plan for the Adhesive Bond

Objective:
Estimate B10 at 50°C with minimum variance.

Constraint:
Maximum testing temperature of 120°C.

Inputs:
Failure probabilities $p_{UL} = 0.001$ and $p_{HL} = 0.90$.

Contour Plot Showing $\log_{10}\{\text{log}(\hat{t}_{1,0.1})/\min \text{Avar}[\log(\hat{t}_{1,0.1})]\}$ as Function of $\xi_L$, $\pi_L$ to Find the Optimum ALT Plan

Adhesive Bond

Weibull Distribution Statistically Optimum Plan

Allocations: $\pi_{Low} = 0.71$ at 95°C, $\pi_{High} = 0.29$ at 120°C

Lognormal Distribution Statistically Optimum Plan

Allocations: $T_{Low} = 71$ at 95°C, $T_{High} = 29$ at 120°C

Simulation of the Weibull Distribution Statistically Optimum Plan

Precision factors $R$ for quantile estimates at 50°C
$R(0.1 \text{ quantile}) = 2.103$
$R(0.5 \text{ quantile}) = 2.309$
$R(Ea) = 1.155$

Results based on 300 simulations
Lines shown for 50 simulations

Adhesive Bond

Statistically Optimum Plan for the Adhesive Bond

• Objective: Estimate B10 at 50°C with minimum variance.
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Adhesive Bond
Lognormal Distribution

Statistically Optimum Plan

- Temp Allocation Failure Expected
  - $\pi_n$ Proportion Number Probability Number Failing
  - $E(\hat{r}_i)$

50.001
78.74 233.13 30
120.26 77.90 69

For this plan and the Lognormal-Arrhenius model, $A_{\log(\hat{t}_{50})} = 0.2002$.

Critique of the Statistically Optimum Plan

- Still too much temperature extrapolation (to 50°C).
- Control the expected number of failures at each experiment.

In general, optimum plans are not robust to model departures.

- Optimum (best) compromise plans—require at least 3 levels of the accelerating variable (e.g., 20%, 50%, 95°C).
- Traditional plans—equal spacing and allocation.
- Optimized plans—maximize statistical precision.

Types of Accelerated Life Test Plans

General Guidelines for Planning ALTs

(Suggested from Optimum Plan Theory)

- Choose the highest level of the accelerating variable to be as high as possible.
- Allocate more units to lower levels of the accelerating varialbe can be optimized.
- Lowest level of the accelerating varialbe can be optimized.

Criteria for Test Planning

Subject to constraints in time, sample size and ranges of experimental variables.

- Maximize the determinant of the Fisher information matrix.
- Minimize $Var[\log(\hat{t}_p)]$ under the assumed model.
- Minimize $Var[\log(\hat{t}_p)]$ (for robustness).
- Choose the highest level of the accelerating variable.

Want a Plan That

- Meets practical constraints and is intuitively appealing.
- Has reasonably good statistical properties.
- Is robust to deviations from assumed inputs.
- Needs practical constraints and intuitively appealing.

General Guidelines for Planning ALTs

(Suggested from Optimum Plan Theory)

- Choose the highest level of the accelerating variable to be as high as possible.
- Lowest level of the accelerating varialbe can be optimized.
- Allocate more units to lower levels of the accelerating varialbe can be optimized.
- Control the expected number of failures at each experiment.

In general, optimum plans not robust to model departures.

- Optimum and the Lognormal-Arrhenius model, $A_{\log(\hat{t}_{50})} = 0.2002$.
- Temp Allocation Failure Expected
  - Proportion Number Probability Number Failing
  - Expected

2002

For this plan and the Lognormal-Arrhenius model, $A_{\log(\hat{t}_{50})} = 0.2002$.

<table>
<thead>
<tr>
<th>Temp</th>
<th>Allocation</th>
<th>Failure</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>70</td>
<td>90</td>
<td>77</td>
</tr>
<tr>
<td>78</td>
<td>74</td>
<td>33</td>
<td>77</td>
</tr>
<tr>
<td>50</td>
<td>01</td>
<td>00</td>
<td>00</td>
</tr>
</tbody>
</table>

Statistically Optimum Plan

Lognormal Distribution
Practical Guidelines for Compromise ALT Plans

• Use three or four levels of the accelerating variable.

• Limit high level of the accelerating variable to maximum reasonable condition.

• Reduce lowest level of the accelerating variable (to minimize extrapolation)—subject to seeing some action.

• Allocate more units to lower levels of the accelerating variable.

• Use statistically optimum plan as a starting point.

• Evaluate plans in various meaningful ways.

Adjusted Compromise Weibull ALT Plan for the Adhesive Bond (20% Constrained Allocation at Middle)

<table>
<thead>
<tr>
<th>Temp</th>
<th>Allocation</th>
<th>Failing</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>76</td>
<td>0.00</td>
<td>120</td>
</tr>
<tr>
<td>Mid</td>
<td>78</td>
<td>0.25</td>
<td>155</td>
</tr>
<tr>
<td>High</td>
<td>80</td>
<td>0.50</td>
<td>200</td>
</tr>
</tbody>
</table>

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Adjusted Compromise Weibull ALT Plan

Simulation of the Adhesive Bond

Compressive on Arrhenius scale

Adhesive Bond

Basic Issue 1: Choose Levels of Accelerating Variables

Need to balance:
• Extrapolation in the acceleration variable (assumed temperature-time relationship).
• Extrapolation in time (assumed failure-time distribution).

Suggested Plan:
• Middle and high levels of the accelerating variable—expect to interpolate in time.
• Low level of the accelerating variable—expect to extrapolate in time.

Basic Issue 2: Allocation of Test Units

• Allocate more test units to low rather than high levels of the accelerating variable.

◮ Tends to equalize the number of failures at experimental conditions.
◮ Testing more units near the use conditions is intuitively appealing.
◮ Suggested by statistically optimum plan.

◮ Need to constrain a certain proportion of units to the mid-level of the accelerating variable.

For this plan with the Weibull-Arrhenius model, $E(a_{50}) = 43.75$.
Properties of Compromise ALT Plans Relative to Statistically Optimum Plans

• Increases asymptotic variance of estimator of \( B_{10} \) at 50\(^\circ\)C by 33% (if assumptions are correct).
• However it also,
• Reduces low test temperature to 78\(^\circ\)C (from 95\(^\circ\)C).
• Uses three levels of accelerating variable, instead of two.
• Is more robust to departures from assumptions and uncertain inputs.

Generalizations and Comments

• Constraints on test positions (instead of test units): Consider replacement after 100\% failures at each level of accelerating variable.
• Continue tests at each level of accelerating variable until at least 100\% units have failed.
• Sequence testing (instead of a single accelerating variable).
• Continue testing at each level of accelerating variable until at least 100\% units have failed.
• Computer simulation.
• Need to quantify robustness.

ALT with Two or More Variables

• Moderate increases in two accelerating variables may be safer than using a large amount of a single accelerating variable.
• There may be interest in assessing the effect of nonaccelerating variables.
• There may be interest in assessing the joint effects of two more accelerating variables.

Choosing Experimental Variable Definition to Minimize Interaction Effects

• Care should be used in defining experimental variables.
• Guidance on variable definition and possible transformations of the response and the experimental models should, as much as possible, be taken from mechanistic models.
• Proper choice can reduce the occurrence or importance of statistical interactions.
• Models without statistical interactions simplify modeling.

Voltage Stress (Volts/mm) versus Size (mm)

<table>
<thead>
<tr>
<th>Volts/mm</th>
<th>Size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>1.5</td>
<td>150</td>
</tr>
<tr>
<td>2.0</td>
<td>200</td>
</tr>
<tr>
<td>2.5</td>
<td>300</td>
</tr>
<tr>
<td>3.0</td>
<td>500</td>
</tr>
</tbody>
</table>

Examples of Choosing Experimental Variable Definition

• For humidity testing of corrosion mechanism use RH and temperature (not vapor pressure and temperature).
• For testing dielectrics, use size and volts stress (e.g., mm and volts).
• For light exposure, use aperture and total light energy.
• To evaluate the adequacy of large-sample approximations with censored data, use % failing and expected number of failures rather than number and exposure time.

Comparison of Experimental Layout with Volts/mm versus Size Versus Volts versus Size

<table>
<thead>
<tr>
<th>Volts</th>
<th>Size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
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<tr>
<td>3.0</td>
<td>500</td>
</tr>
</tbody>
</table>
Comparison of Experimental Layout with
Volts versus Size and Volts/mm versus Size

Volts

<table>
<thead>
<tr>
<th>Size (mm)</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1.5</td>
<td>...</td>
<td>...</td>
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<td>...</td>
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<tr>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<td>...</td>
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<td>...</td>
</tr>
<tr>
<td>3.0</td>
<td>...</td>
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<td>...</td>
</tr>
</tbody>
</table>

Voltage Stress

[Volts/mm]

<table>
<thead>
<tr>
<th>Size (mm)</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>1.5</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>2.0</td>
<td>...</td>
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<td>...</td>
</tr>
<tr>
<td>2.5</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>3.0</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Insulation ALT

From Chapter 6 of Nelson (1990) and Escobar and Meeker (1995)

- Engineers needed rapid assessment of insulation life at use conditions.
- 1000/10000 hours available for testing.
- 170 test units available for testing.
- Possible experimental variables:
  - VPM (Volts/mm) [accelerating].
  - THICK (cm) [nonaccelerating].
  - TEMP (°C) [accelerating].

Models Used in Examples

- \( \mu = \beta_0 + \beta_1 \log(\text{VPM}) \)
- \( \mu = \beta_0 + \beta_1 \log(\text{VPM}) + \beta_2 \log(\text{THICK}) \)
- \( \mu = \beta_0 + \beta_1 \log(\text{VPM}) + \beta_2 [11605 + 273 \cdot \text{TEMP}] \)

Models

1. [\text{ALT Design Problem}]

- Design test plan to estimate life at the use conditions of \( \text{VPM} = 80 \) volts/mm, \( \text{THICK} = 0 \) cm, \( \text{TEMP} = 120 \) °C.

Model and Assumptions

- Observations statistically independent.
- Units tested simultaneously until censoring time.
- Failure-time distribution.
- Does not depend on the experimental variables.
- \( \mu(x) \) a function of the accelerating (or other experimental) variables.
- \( \sigma \) constant.

Multiple Variable ALT

Conditions

- 3 × 3 VPM × THICK Factorial Test Plan

The ALT Design Problem

- Interest centers on a quantile in lower tail of life distribution.

- Need to choose levels of the accelerating variables.

Design test plan to estimate life at the use conditions of \( \text{VPM} \), \( \text{THICK} \), \( \text{TEMP} \) conditions.

- 170 test units available for testing.
- 1000/10000 hours available for testing.

Conditions

- Engineers needed rapid assessment of insulation life at use conditions.
- From Chapter 6 of Nelson (1990) and Escobar and Meeker (1995)
Multi-Variable Experimental Region

• Maximum levels for all variables:
  - VPM \( H = 200 \) volts/mm
  - THICK \( H = 0.355 \) cm
  - TEMP \( H = 260 \) °C.

• Explicit minimum levels for all experimental variables:
  - VPM \( A = 80 \) volts/mm
  - THICK \( A = 0.163 \) cm
  - TEMP \( A = 120 \) °C (also stricter implicit limits for VPM and TEMP).

• May need to restrict highest combinations of accelerating variables; e.g., constrain by equal failure-probability line (by using a maximum failure probability constraint \( p^\ast \) or equivalently a standardized censored failure time constraint \( \zeta^\ast \)).

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Insulation ALT

VPM × THICK Optimum Test Plan

-0.2 0.0 0.2 0.4 0.6 0.8 1.0 1.2

Standardized Factor 1

-2 -1 0 1

Standardized Factor 2

80 100 120 140 160 180 200

Electrical Stress (volts per mm)

0.15 0.2 0.25 0.3 0.35 0.4

Thickness (cm)

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p = U = 1.8 \times 10^{-6}

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Optimum Degenerate Plan: Technical Results

• When acceleration does not help sufficiently, it is optimum to test all units at the use conditions.

• Otherwise there is at least one optimum degenerate test plan in the \( x_1 \times x_2 \) plane.

• Some units tested at highest levels of accelerating variables.

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Splitting Degenerate Plans

• It is possible to split a degenerate plan into a nondegenerate plan.

• Use secondary criteria to choose best split plan.

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Optimum Test Plan with \( p^\ast / \zeta^\ast \) constraint

-0.2 0.0 0.2 0.4 0.6 0.8 1.0 1.2

Standardized Factor 1

-2 -1 0 1

Standardized Factor 2

80 100 120 140 160 180 200

Electrical Stress (volts per mm)

0.15 0.2 0.25 0.3 0.35 0.4

Thickness (cm)

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Degenerate and Nondegenerate Test Plans

Optimum degenerate plan corresponds to a single-variable optimum plan.

Some units tested at highest levels of accelerating variables.

Othervise there is at least one optimum degenerate test plan.

Otherwise there is at least one optimum degenerate test plan.

When acceleration does not help sufficiently, it is optimum plan.

Test all units at the use conditions.

Test all units at the use conditions.

Test two (or more) combinations of the experimental variables on a line with slope \( s \) passing through \( x_u \).

Test at three (or more) noncollinear combinations of the experimental variables in the plane.

Estimate \( \delta \).

Degenerate plans:

• Test all units at \( x_u \).

Nondegenerate practical plans:

• Test at three (or more) noncollinear combinations of the experimental variables in the plane.
Comparison of Test Plan Properties

Optimum Test Plan

Insulation VPM x TEMP

3x3 Factorial Test Plan

Insulation VPM x TEMP

20% Compromise Test Plan with \(p/\sqrt{c}\) constant

Comparison of Test Plan Properties

Optimum Test Plan

Insulation VPM x TEMP

3x3 Factorial Test Plan

Insulation VPM x TEMP

20% Compromise Test Plan with \(p/\sqrt{c}\) constant

Comparison of Test Plan Properties
• With one accelerating and several other regular experimental variables, replicate single-variable ALT at each combination of the regular experimental variables.

• Can use a fractional factorial for the regular experimental variables.

• If the approximate effect of a regular experimental variable is known, can tilt factorial to improve precision.

• With two or more accelerating variables, our results show how to tilt the traditional factorial plans to restrict extrapolation and maintain statistical efficiency.

Extensions of Results to Other Problems