Chapter 19

Analyzing Accelerated Life Test Data

William Q. Meeker and Luis A. Escobar
Iowa State University and Louisiana State University

Based on the authors' text *Statistical Methods for Reliability Data*, John Wiley & Sons Inc. 1998.

July 30, 2009
8h 5min

Chapter 19
Analyzing Accelerated Life Test Data
Objectives

- Describe and illustrate nonparametric and graphical methods of analyzing and presenting accelerated life test data.
- Describe and illustrate maximum likelihood methods of analyzing and making inferences from accelerated life test data.
- Illustrate different kinds of data and ALT models.
- Discuss some specialized applications of accelerated testing.
- Describe pitfalls in accelerated testing.

Example: Temperature-Accelerated Life Test on Device-A (from Hooper and Amster 1990)

Data: Singly right censored observations from a temperature-accelerated life test.

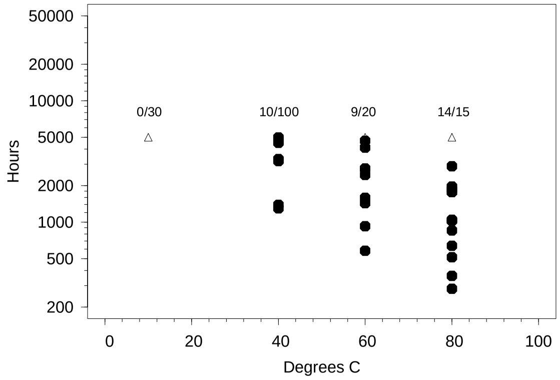
Purpose: To determine if the device would meet its hazard function objective at 10,000 and 30,000 hours at operating temperature of 10°C.

We will show how to fit an accelerated life regression model to these data to answer this and other questions.

Hours Versus Temperature Data from a Temperature-Accelerated Life Test on Device-A

Hours	Status	Number of Devices	Temperature °C	In Subexperiment	
				Units	Failures
5000	Censored	30	10	30	0/30
1298	Failed	1	40	100	10/100
1390	Failed	1	40		
⋮	⋮	⋮	⋮		
5000	Censored	90	40		
581	Failed		60	20	9/20
925	Failed		60		
1432	Failed		60		
⋮	⋮	⋮	⋮		
5000	Censored	11	60		
283	Failed	1	80	15	14/15
361	Failed	1	80		
515	Failed	1	80		
638	Failed	1	80		
⋮	⋮	⋮	⋮		
5000	Censored	1	80		

Device-A Hours Versus Temperature (Hooper and Amster 1990)



ALT Data Plot

- Examine a scatter plot of lifetime versus stress data.
- Use different symbols for censored observations.

Note: Heavy censoring makes it difficult to identify the form of the life/stress relationship from this plot.

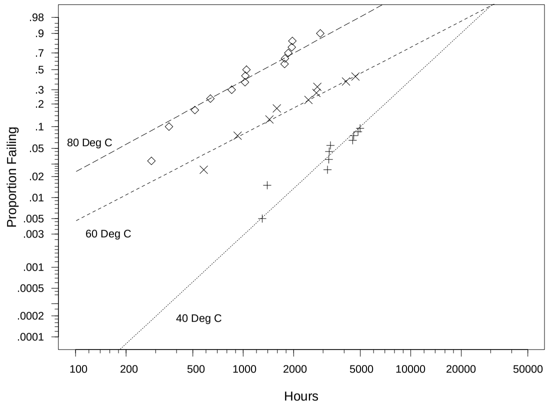
Strategy for Analyzing ALT Data

For ALT data consisting of a number of subexperiments, each having been run at a particular set of conditions:

- Examine the data graphically: Scatter and probability plots.
- Use a multiple probability plot to study the data from the individual subexperiments.
- Fit an overall model involving a life/stress relationship.
- Perform residual analysis and other diagnostic checks.
- Perform a sensitivity analysis.
- Assess the reasonableness of using the ALT data to make the desired inferences.

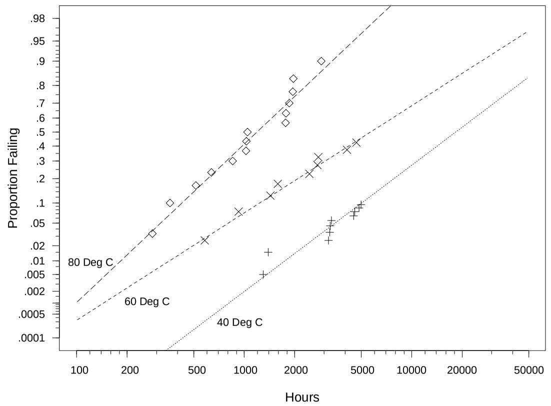
Weibull Multiple Probability Plot Giving Individual Weibull Fits to Each Level of Temperature for Device-A ALT Data

$$\widehat{\Pr}\left[T(\texttt{temp}_i) \le t\right] = \Phi_{\text{sev}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}_i}\right], \quad i = 40, 60, 80$$



Lognormal Multiple Probability Plot Giving Individual Lognormal Fits to Each Level of Temperature for Device-A ALT Data

$$\widehat{\Pr}\left[T(\texttt{temp}_i) \le t\right] = \Phi_{\text{nor}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}_i}\right], \quad i = 40, 60, 80$$



ALT Multiple Probability Plot of Nonparametric Estimates at Individual Levels of Accelerating Variable

- Compute nonparametric estimates $\widehat{F}$ for each level of accelerating variable; plot on a single probability plot.
- Try to identify a distributional model that fits the data well at all of the stress-levels.

  Note: Either the lognormal or the Weibull distribution model provides a reasonable description for the device-A data. But the lognormal distribution provides a better fit to the individual subexperiments.

ALT Multiple Probability Plot of ML Estimates at Individual Levels of Accelerating Variable

- For each **individual** level of accelerating variable compute the ML estimates.

  Let T_i be the failure time at temperature Temp$_i$. For the **lognormal**, $T_i \sim \text{LOGNOR}(\mu_i, \sigma_i)$, assumed model:

  - ▶ Compute ML estimates $(\widehat{\mu}_i, \widehat{\sigma}_i)$.
  - ▶ Plot the LOGNOR$(\widehat{\mu}_i, \widehat{\sigma}_i)$ cdfs on same plot.
- Assess the commonly used assumption that σ_i does not depend on Temp$_i$ and that Temp$_i$ only affects μ_i.

  Note: There are some small differences among the slopes that could be due to sampling error.

Device-A ALT Lognormal ML Estimation Results at Individual Temperatures

	Parameter	ML Estimate	Standard Error	95% Approximate Confidence Interval Lower	Upper
40°C	μ	9.81	.42	8.9	10.6
	σ	1.0	.27	.59	1.72
60°C	μ	8.64	.35	8.0	9.3
	σ	1.19	.32	.70	2.0
80°C	μ	7.08	.21	6.7	7.5
	σ	.80	.16	.55	1.17

The individual loglikelihoods were $\mathcal{L}_{40} = -115.46$, $\mathcal{L}_{60} = -89.72$, and $\mathcal{L}_{80} = -115.58$. The confidence intervals are based on the normal approximation method.

The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

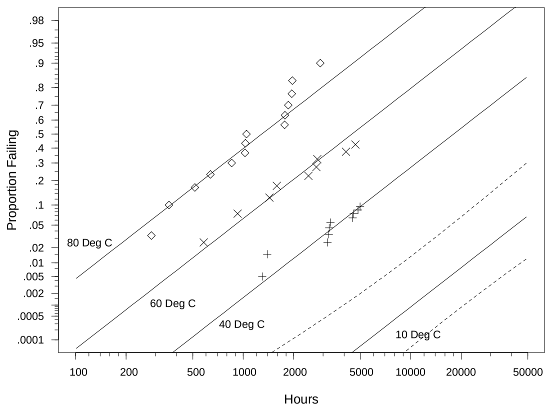
$$\Pr[T(\texttt{temp}) \le t] = \Phi_{\text{nor}}\left[\frac{\log(t) - \mu(x)}{\sigma}\right]$$

where

- $\mu(x) = \beta_0 + \beta_1 x$,
- $x = 11605/(\texttt{temp}\,\text{K}) = 11605/(\texttt{temp}\,°\text{C} + 273.15)$,
- $\beta_1 = E_a$ is the activation energy, and
- σ assumed to be constant.

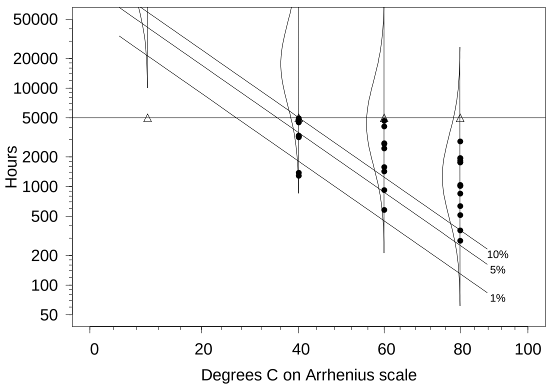
Lognormal Multiple Probability Plot of the Arrhenius-Lognormal Log-Linear Regression Model Fit to the Device-A ALT Data

$$\widehat{\Pr}\,[T(\texttt{temp}) \le t] = \Phi_{\text{nor}}\left[\frac{\log(t) - \widehat{\mu}(x)}{\widehat{\sigma}}\right], \quad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



Scatter plot showing the Arrhenius-Lognormal Log-Linear Regression Model Fit to the Device-A ALT Data

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\text{nor}}(p)\widehat{\sigma}, \quad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



ML Estimation Results for the Device-A ALT Data and the Arrhenius-Lognormal Regression Model

	ML	Standard	95% Approximate Confidence Intervals	
Parameter	Estimate	Error	Lower	Upper
β_0	−13.5	2.9	−19.1	−7.8
β_1	.63	.08	.47	.79
σ	.98	.13	.75	1.28

The loglikelihood is $\mathcal{L} = -321.7$. The confidence intervals are based on the normal approximation method.

Analytical Comparison of Individual and Arrhenius-Lognormal Model ML Estimates of Device-A Data

- Distributions fit to individual levels of temperature can be viewed as an **unconstrained model**.
- The Arrhenius-lognormal regression model can be viewed as a **constrained** model (μ linear in x and σ constant).
- Use likelihood ratio test to check for lack of fit with respect to the constraints.

$$\begin{aligned} \mathcal{L}_{\text{unconst}} &= \mathcal{L}_{40} + \mathcal{L}_{60} + \mathcal{L}_{80} = -320.76 \\ \mathcal{L}_{\text{const}} &= -321.7 \end{aligned}$$

- $-2(\mathcal{L}_{\text{const}} - \mathcal{L}_{\text{unconst}}) = -2(-321.7 + 320.76) = 1.88 < \chi^2_{(.75,3)} = 4.1$, indicating that there is no evidence of inadequacy of the constrained model, relative to the unconstrained model.

ALT Multiple Probability Plot of ML Estimates with an Assumed Life/Stress Relationship

- To make inferences at levels of accelerating variable not used in the ALT, use a life/stress relationship to fit all the data.

  Let $T(x_i)$ be the failure time at $x_i = 11605/(\text{Temp}_i + 273.15)$. For the, $T(x_i) \sim \text{LOGNOR}(\mu(x_i) = \beta_0 + \beta_1 x_i, \sigma)$, **lognormal** SAFT assumed model:

  - ▶ Compute ML estimates $(\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\sigma})$.
  - ▶ Plot the LOGNOR$\left[\widehat{\mu}(x_i) = \widehat{\beta}_0 + \widehat{\beta}_1 x_i, \widehat{\sigma}\right]$ cdfs on same plot.
  - ▶ Plot $\widehat{t}_p(x) = \exp\left[\widehat{\beta}_0 + \widehat{\beta}_1 x + \Phi^{-1}_{\text{nor}}(p)\widehat{\sigma}\right]$ for various values of p and a range of values of x.

ML Estimation for the Device-A Lognormal Distribution F(30,000) at 10°C

$$\widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$
$$= -13.469 + .6279 \times 11605/(10 + 273.15) = 12.2641$$
$$\widehat{\zeta}_e = [\log(t_e) - \widehat{\mu}]/\widehat{\sigma} = [\log(30{,}000) - 12.2641]/.9778$$
$$= -2.000$$
$$\widehat{F}(30{,}000) = \Phi_{\text{nor}}(\widehat{\zeta}_e) = \Phi_{\text{nor}}(-2.000) = .02281$$

$$\widehat{\Sigma}_{\widehat{\mu},\widehat{\sigma}} = \begin{bmatrix} \widehat{\text{Var}}(\widehat{\mu}) & \widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) \\ \widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) & \widehat{\text{Var}}(\widehat{\sigma}) \end{bmatrix} = \begin{bmatrix} .287 & .048 \\ .048 & .0176 \end{bmatrix}.$$

$$\widehat{\text{se}}_{\widehat{F}} = \frac{\phi(\widehat{\zeta}_e)}{\widehat{\sigma}} \left[\widehat{\text{Var}}(\widehat{\mu}) + 2\widehat{\zeta}_e \widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) + \widehat{\zeta}_e^{\,2} \widehat{\text{Var}}(\widehat{\sigma}) \right]^{\frac{1}{2}}$$
$$= \frac{\phi(-2.000)}{.9778} \left[.286 + 2 \times (-2.000) \times .047 + (-2.000)^2 \times .0176 \right.$$
$$= .0225.$$

Confidence Interval for the Device-A Lognormal Distribution F(30,000) at 10°C

A 95% normal-approximation confidence interval based on the assumption that $Z_{\text{logit}(\widehat{F})} \sim \text{NOR}(0,1)$ is

$$[\underset{\sim}{F}(t_e), \quad \widetilde{F}(t_e)] = \left[\frac{\widehat{F}}{\widehat{F} + (1-\widehat{F}) \times w}, \quad \frac{\widehat{F}}{\widehat{F} + (1-\widehat{F})/w} \right]$$
$$= \left[\frac{.02281}{.02281 + (1-.02281) \times w}, \quad \frac{.02281}{.02281 + (1-.02281)/w} \right]$$
$$= [.0032, \quad .14]$$

where

$$w = \exp\{(z_{(1-\alpha/2)} \widehat{\text{se}}_{\widehat{F}})/[\widehat{F}(1-\widehat{F})]\}$$
$$= \exp\{(1.96 \times .0225)/[.02281(1-.02281)]\} = 7.232.$$

This wide interval reflects sampling uncertainty when activation energy is unknown. The interval does not reflect model uncertainty. With given activation energy, the confidence intervals would be much narrower.

Checking Model Assumptions

It is important to check model assumptions by using residual analysis and other model diagnostics

- Define standardized residuals as

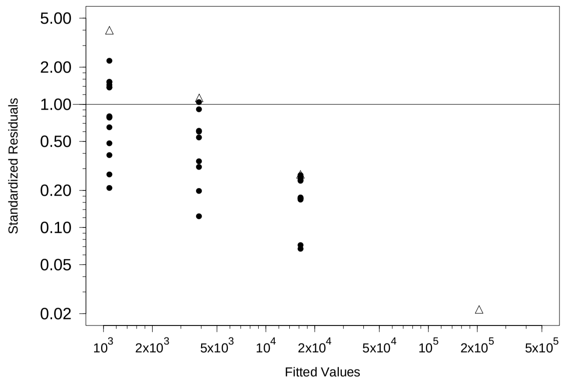
$$\exp\left\{ \frac{\log[t(x_i)] - \widehat{\beta}_0 - \widehat{\beta}_1 x_i}{\widehat{\sigma}} \right\}$$

where $t(x_i)$ is a failure time at x_i.

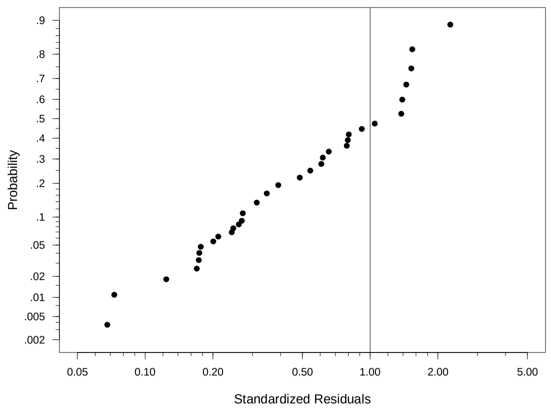
- Residuals corresponding to censored observations are called **censored** standardized residuals.
- Plot residuals versus the fitted values given by $\exp\left(\widehat{\beta}_0 + \widehat{\beta}_1 x_i\right)$.
- Do a probability plot of the residuals.

Note: For the Device-A data, these plots do not conflict with the model assumptions.

Plot of Standardized Residuals Versus Fitted Values for the Arrhenius-Lognormal Log-Linear Regression Model Fit to the Device-A ALT Data



Probability Plot of the Residuals from the Arrhenius-Lognormal Log-Linear Regression Model fit to the Device-A ALT Data



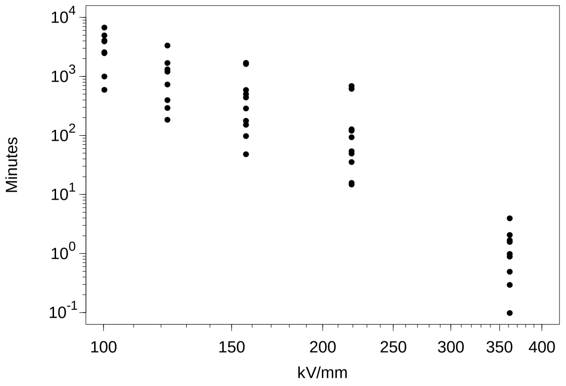
Some Practical Suggestions

- Build on previous experience with similar products and materials.
- Use pilot experiments; evaluate the effect of stress on degradation and life.
- Seek physical understanding of cause of failure.
- Use results from physical failure mode analysis.
- Seek physical justification for life/stress relationships.
- Design tests to limit the amount extrapolation needed for desired inferences.
- See Nelson (1990).

Inferences from AT Experiments

- Inferences or predictions from ATs require important assumptions about:
  - ▶ Focused correctly on relevant failure modes.
  - ▶ Adequacy of AT model for extrapolation.
  - ▶ AT manufacturing testing processes can be related to actual manufacturing/use of product.
- Important sources of variability usually overlooked.
- Deming would call ATs **analytic studies** (see Hahn and Meeker 1993, *American Statistician*).

Breakdown Times in Minutes of a Mylar-Polyurethane Insulating Structure (from Kalkanis and Rosso 1989)

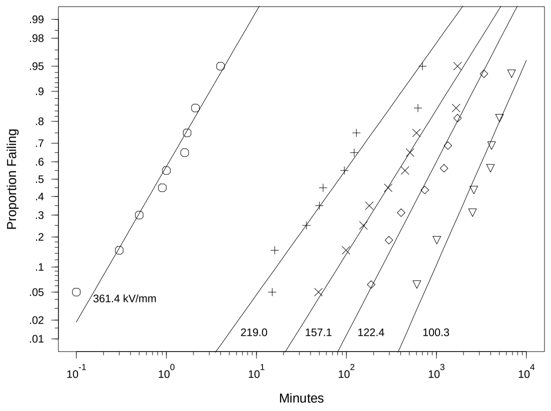


Accelerated Life Test of a Mylar-Polyurethane Laminated Direct Current High Voltage Insulating Structure

- Data from Kalkanis and Rosso (1989)
- Time to dielectric breakdown of units tested at 100.3, 122.4, 157.1, 219.0, and 361.4 kV/mm.
- Needed to evaluate the reliability of the insulating structure and to estimate the life distribution at system design voltages (e.g. 50 kV/mm).
- Except for the highest level of voltage, the relation between log life and log voltage appears to be approximately linear.
- Failure mechanism probably different at 361.4 kV/mm.

Lognormal Probability Plot of the Individual Tests in the Mylar-Polyurethane ALT

$$\widehat{\Pr}\left[T(\texttt{temp}_i) \le t\right] = \Phi_{\text{nor}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}_i}\right], \quad i = 100.3, \ldots, 361.4$$



Inverse Power Relationship-Lognormal Model

- The inverse power relationship-lognormal model is

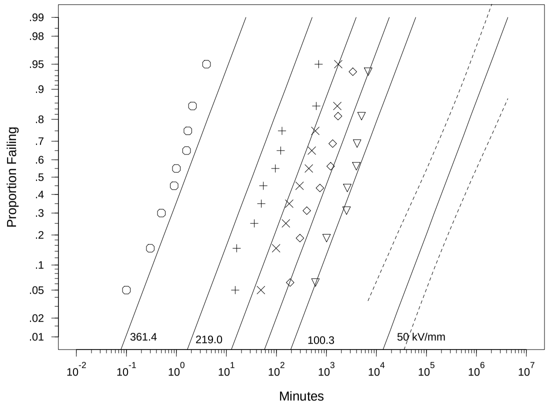
$$F(t) = \Pr[T(\texttt{volt}) \le t] = \Phi_{\text{nor}}\left[\frac{\log(t) - \mu(x)}{\sigma}\right]$$

where $\mu(x) = \beta_0 + \beta_1 x$, and $x = \log(\text{Voltage Stress})$.

- σ assumed to be constant.

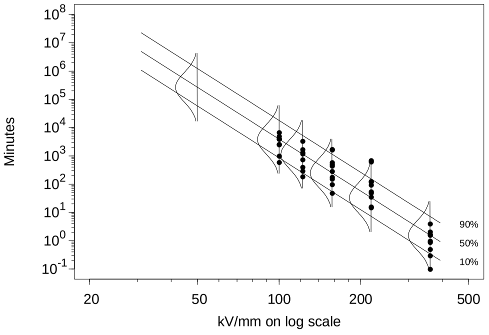
Lognormal Probability Plot of the Inverse Power Relationship-Lognormal Model Fitted to the Mylar-Polyurethane Data Including 361.4 kV/mm

$$\widehat{\Pr}\left[T(\texttt{temp}_i) \le t\right] = \Phi_{\text{nor}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}_i}\right], \quad i = 100.3, \ldots, 361.4$$

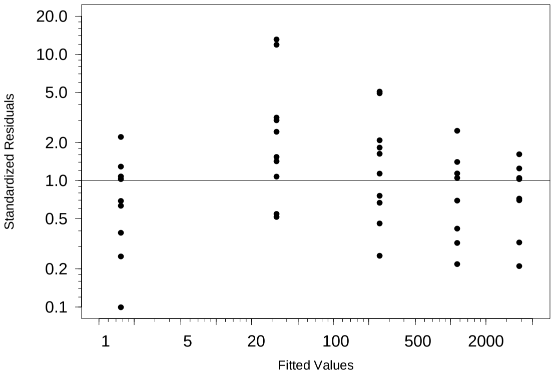


Plot of Inverse Power Relationship-Lognormal Model Fitted to the Mylar-Polyurethane Data Including 361.4 kV/mm

$$\log[\hat{t}_p(x)] = \hat{\mu}(x) + \Phi^{-1}_{\text{nor}}(p)\hat{\sigma}, \quad \hat{\mu}(x) = \hat{\beta}_0 + \hat{\beta}_1 x$$

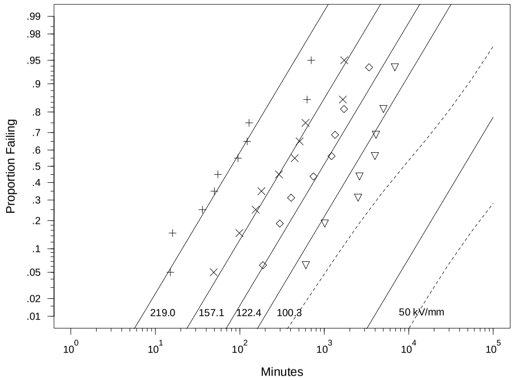


Lognormal Plot of the Standardized Residuals versus $\exp(\hat{\mu}(x))$ for the Inverse Power Relationship-Lognormal Model Fitted to the Mylar-Polyurethane Data with the 361.4 kV/mm Data



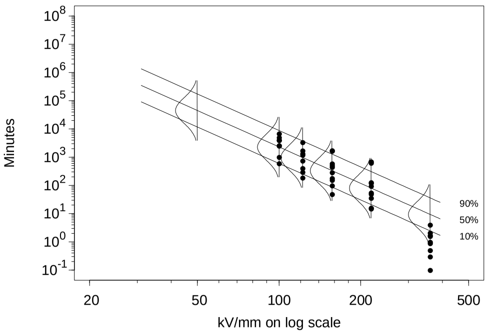
Lognormal Probability Plot of the Inverse Power Relationship-Lognormal Model Fitted to the Mylar-Polyurethane Data W/O the 361.4 kV/mm Data

$$\widehat{\Pr}\left[T(\texttt{temp}) \le t\right] = \Phi_{\text{nor}}\left[\frac{\log(t) - \hat{\mu}(x)}{\hat{\sigma}}\right], \quad \hat{\mu}(x) = \hat{\beta}_0 + \hat{\beta}_1 x$$



Plot of Inverse Power Relationship-Lognormal Model Fitted to the Mylar-Polyurethane Data (also Showing 361.4 kV/mm Data Omitted from the ML Estimation)

$$\log[\hat{t}_p(x)] = \hat{\mu}(x) + \Phi^{-1}_{\text{nor}}(p)\hat{\sigma}, \quad \hat{\mu}(x) = \hat{\beta}_0 + \hat{\beta}_1 x$$

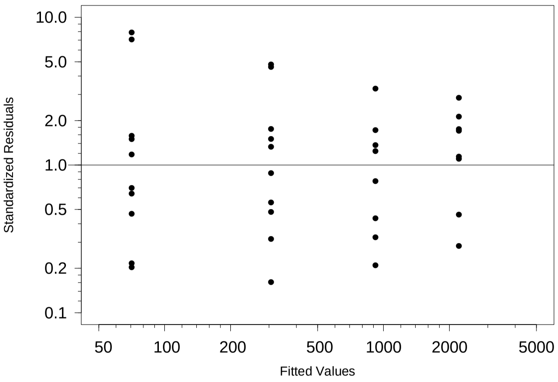


Inverse Power Relationship-Lognormal Model ML Estimation Results for the Mylar-Polyurethane ALT Data

	ML	Standard	95% Approximate Confidence Intervals	
Parameter	Estimate	Error	Lower	Upper
β_0	27.5	3.0	21.6	33.4
β_1	−4.29	.60	−5.46	−3.11
σ	1.05	.12	.83	1.32

The loglikelihood is $\mathcal{L} = -271.4$. The confidence intervals are based on the normal approximation method.

Lognormal Plot of the Standardized Residuals versus $\exp(\hat{\mu})$ for the Inverse Power Relationship-Lognormal Model Fitted to the Mylar-Polyurethane Data W/O the 361.4 kV/mm Data



Analysis of Interval ALT Data on a New-Technology IC Device

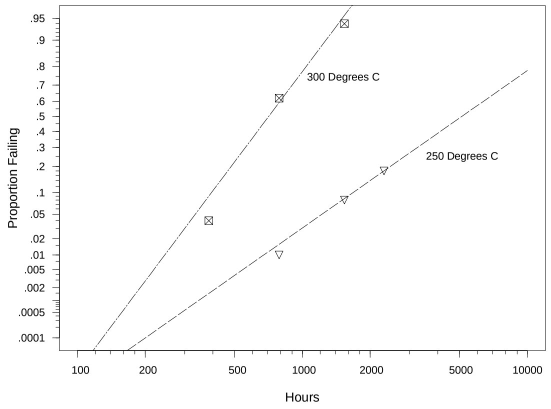
- Tests were run at 150, 175, 200, 250, and 300°C.
- Developers interested in estimating activation energy of the suspected failure mode and the long-life reliability.
- Failures had been found only at the two higher temperatures.
- After early failures at 250 and 300°C, there was some concern that no failures would be observed at 175°C before decision time.
- Thus the 200°C test was started later than the others.

New-Technology IC Device ALT Data

Hours Lower	Upper	Status	Number of Devices	Temperature °C
	1536	Right Censored	50	150
	1536	Right Censored	50	175
	96	Right Censored	50	200
384	788	Failed	1	250
788	1536	Failed	3	250
1536	2304	Failed	5	250
	2304	Right Censored	41	250
192	384	Failed	4	300
384	788	Failed	27	300
788	1536	Failed	16	300
	1536	Right Censored	3	300

Lognormal Probability Plot of the Failures at 250 and 300°C for the New-Technology Integrated Circuit Device ALT Experiment

$$\widehat{\Pr}\left[T(\texttt{temp}_i) \le t\right] = \Phi_{\text{nor}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}_i}\right], \quad i = 250, 300$$



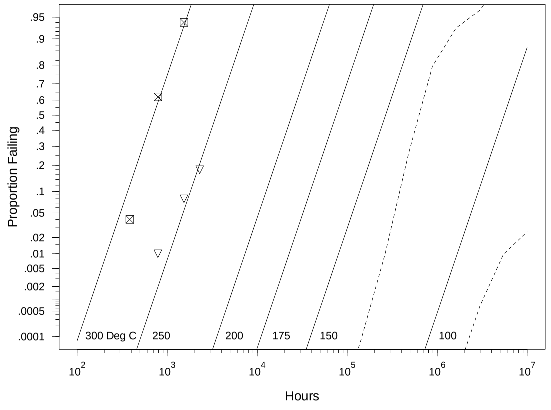
Individual Lognormal ML Estimation Results for the New-Technology IC Device

	Parameter	ML Estimate	Standard Error	95% Approximate Confidence Intervals Lower	Upper
250°C	μ	8.54	.33	7.9	9.2
	σ	.87	.26	.48	1.57
300°C	μ	6.56	.07	6.4	6.7
	σ	.46	.05	.36	.58

The loglikelihood were $\mathcal{L}_{250} = -32.16$ and $\mathcal{L}_{300} = -53.85$. The confidence intervals are based on the normal approximation method.

Lognormal Probability Plot Showing the Arrhenius-Lognormal Model ML Estimation Results for the New-Technology IC Device

$$\widehat{\Pr}\left[T(\texttt{temp}) \le t\right] = \Phi_{\text{nor}}\left[\frac{\log(t)-\widehat{\mu}(x)}{\widehat{\sigma}}\right], \quad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



Arrhenius-Lognormal Model ML Estimation Results for the New-Technology IC Device

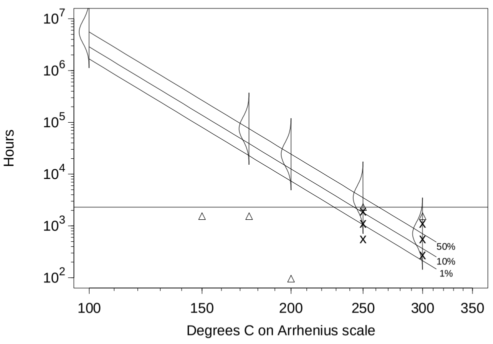
Parameter	ML Estimate	Standard Error	95% Approximate Confidence Intervals Lower	Upper
β_0	−10.2	1.5	−13.2	−7.2
β_1	.83	.07	.68	.97
σ	.52	.06	.42	.64

The loglikelihood is $\mathcal{L} = -88.36$.

Comparing the constrained and unconstrained models $\mathcal{L}_{\text{uconst}} = \mathcal{L}_{250} + \mathcal{L}_{300} = -86.01$ and for the constrained model, $\mathcal{L}_{\text{const}} = -88.36$. The comparison has just one degree of freedom and $-2(-88.36 + 86.01) = 4.7 > \chi^2_{(.95,1)} = 3.84$, again indicating that there is some lack of fit in the constant-σ Arrhenius-lognormal model.

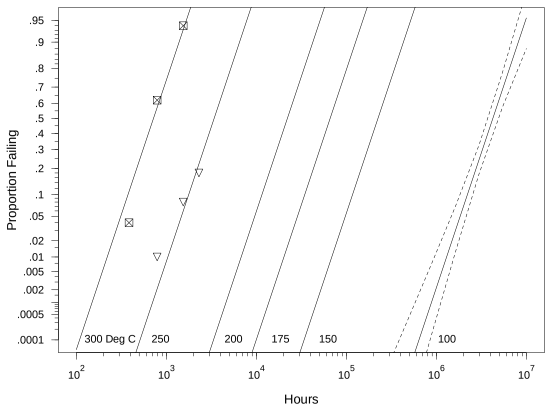
Arrhenius Plot Showing ALT Data and the Arrhenius-Lognormal Model ML Estimation Results for the New-Technology IC Device.

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\text{nor}}(p)\widehat{\sigma}, \quad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



Lognormal Probability Plot Showing the Arrhenius-Lognormal Model ML Estimation Results for the New-Technology IC Device with Given $E_a = .8$

$$\widehat{\Pr}\left[T(\texttt{temp}) \le t\right] = \Phi_{\text{nor}}\left[\frac{\log(t) - \widehat{\mu}(x)}{\widehat{\sigma}}\right], \quad \widehat{\mu}(x) = \widehat{\beta}_0 + E_a x$$



Pitfall 1: Multiple (Unrecognized) Failure Modes

- High levels of accelerating factors can induce failure modes that would not be observed at normal operating conditions (or otherwise change the life-acceleration factor relationship).
- Other failure modes, if not recognized in data analysis, can lead to incorrect conclusions.
- Suggestions:
  - Determine failure mode of failing units.
  - Analyze failure modes separately.

Pitfall 2: Failure to Properly Quantify Uncertainty

- There is uncertainty in statistical estimates.
- Standard statistical confidence intervals quantify uncertainty arising from **limited data**.
- Confidence intervals **ignore model uncertainty** (which can be tremendously amplified by extrapolation in Accelerated Testing).
- Suggestions:
  - Use confidence intervals to quantify statistical uncertainty.
  - Use sensitivity analysis to assess the effect of departures from model assumptions and uncertainty in other inputs.

Pitfall 3: Multiple Time Scales

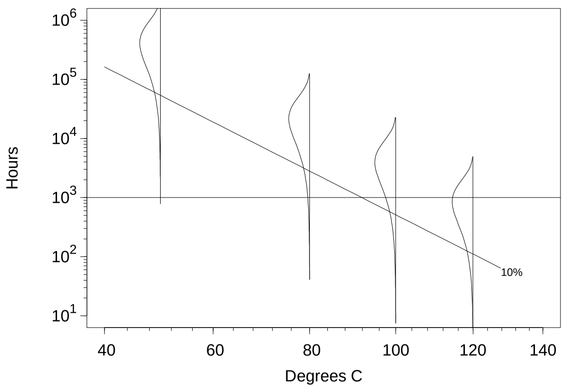
- Composite material
  - Chemical degradation over time changes material ductility.
  - Stress cycles during use lead to initiation and growth of cracks.
- Incandescent light bulbs
  - Filament evaporates during burn time.
  - On-off cycles induce thermal and mechanical shocks that can lead to fatigue cracks.
- Inkjet pen
  - Real time (corrosion)
  - Characters or pages printed (ink supply, resistor degradation).
  - On/off cycles of a print operation (thermal cycling of substrate and printhead lamination).

Dealing with Multiple Time Scales

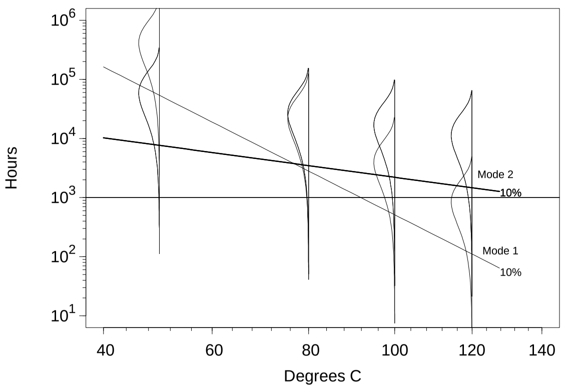
Suggestions:

- Need to use the appropriate time scale(s) for evaluation of each failure mechanism.
- With multiple time scales, understand ratio or ratios of time scales that arise in actual use.
- Plan ATs that will allow effective prediction of failure time distributions at desired ratio (or ratios) of time scales.

Temperature-Accelerated Life Test for an IC Device



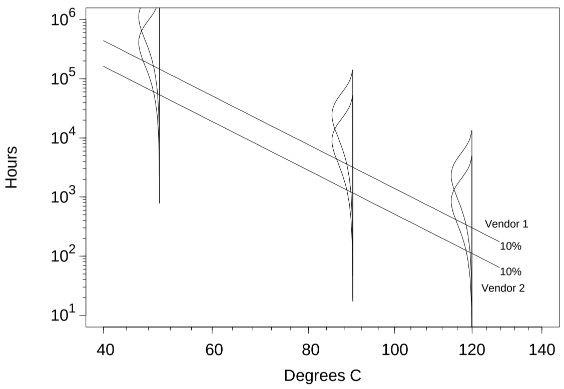
Unmasked Failure Mode with Lower Activation Energy for an IC Device



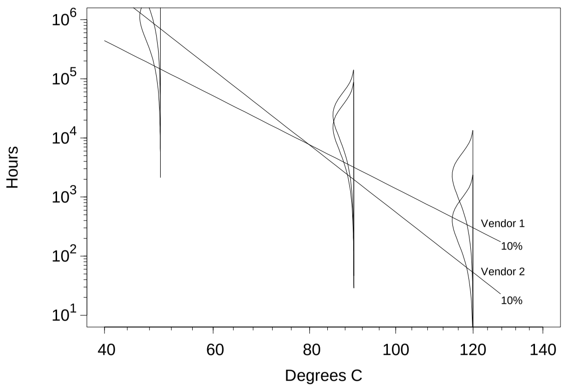
Pitfall 4: Masked Failure Mode

- Accelerated test may focus on one known failure mode, masking another!
- Masked failure modes may be the first one to show up in the field.
- Masked failure modes could dominate in the field.
- Suggestions:
  - ▶ Know (anticipate) different failure modes.
  - ▶ Limit acceleration and test at levels of accelerating variables such that each failure mode will be observed at two or more levels of the accelerating variable.
  - ▶ Identify failure modes of all failures.
  - ▶ Analyze failure modes separately.

Comparison of Two Products I Simple Comparison



Comparison of Two Products II Questionable Comparison



Pitfall 5: Faulty Comparison

- It is sometimes claimed that Accelerated Testing is not useful for predicting reliability, but is useful for comparing alternatives.
- Comparisons can, however, also be misleading.
- Beware of comparing products that have different kinds of failures.
- Suggestions:
  - ▶ Know (anticipate) different failure modes.
  - ▶ Identify failure modes of all failures.
  - ▶ Analyze failure modes separately.
  - ▶ Understand the physical reason for any differences.

Pitfall 6: Acceleration Factors Can Cause Deceleration!

- Increased temperature in an **accelerated** circuit-pack reliability audit resulted in fewer failures than in the field because of lower humidity in the **accelerated** test.
- Higher than usual use rate of a mechanical device in an accelerated test inhibited a corrosion mechanism that eventually caused serious field problems.
- Automobile air conditioners failed due to a material **drying out** degradation, lack of use in winter (not seen in continuous accelerated testing).
- Inkjet pens fail from infrequent use.
- **Suggestion**: Understand failure mechanisms and how they are affected by experimental variables.

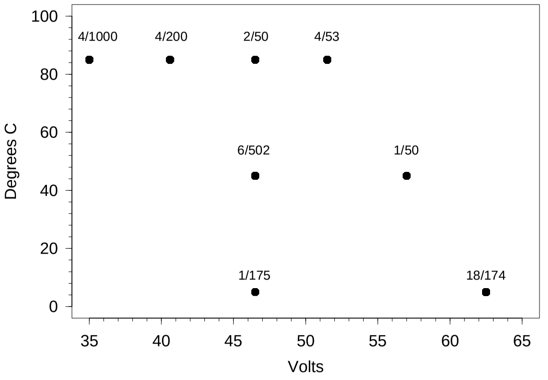
Pitfall 7: Untested Design/Production Changes

- Lead-acid battery cell designed for 40 years of service.
- New epoxy seal to inhibit **creep** of electrolyte up the positive post.
- Accelerated life test described in published article **demonstrated** 40 year life under normal operating conditions.
- 200,000 units in service after 2 years of manufacturing.
- First failure after 2 years of service; third and fourth failures followed shortly thereafter.
- Improper epoxy cure combined with charge/discharge cycles hastened failure.
- Entire population had to be replaced with a re-designed cell.

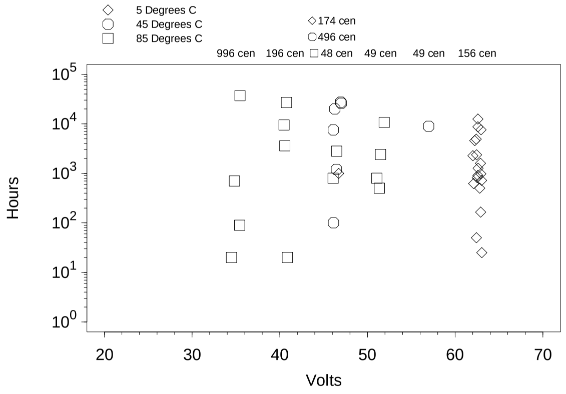
Temperature/Voltage ALT Data on Tantalum Electrolytic Capacitors

- Two-factor ALT
- Non-rectangular unbalanced design
- Much censoring
- The Weibull distribution seems to provide a reasonable model for the failures at those conditions with enough failures to make a judgment.
- Temperature effect is not as strong.
- Data analyzed in Singpurwalla, Castellino, and Goldschen (1975)

Tantalum Capacitors ALT Design Showing Fraction Failing at Each Point

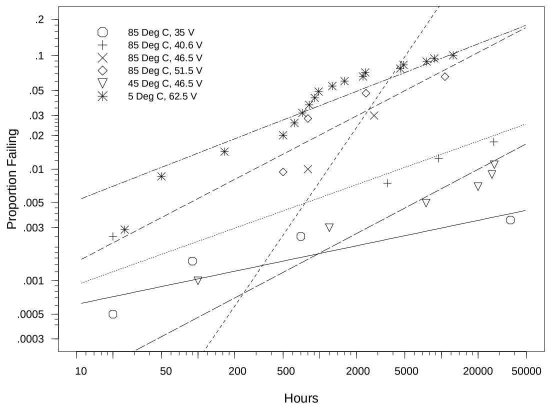


Scatter Plot of Failures in the Tantalum Capacitors ALT Showing Hours to Failure Versus Voltage with Temperature Indicated by Different Symbols



Weibull Probability Plot for the Individual Voltage and Temperature Level Combinations for the Tantalum Capacitors ALT, with ML Estimates of Weibull cdfs

$$\widehat{\Pr}\left[T(\texttt{temp}_i) \le t\right] = \Phi_{\text{sev}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}_i}\right]$$



Tantalum Capacitors ALT
Weibull/Arrhenius/Inverse Power Relationship Models

$$\begin{aligned}\text{Model 1:}\quad \mu(\boldsymbol{x}) &= \beta_0+\beta_1x_1+\beta_2x_2\\ \text{Model 2:}\quad \mu(\boldsymbol{x}) &= \beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_1x_2\end{aligned}$$

where $x_1=\log(\texttt{volt})$, $x_2=11605/(\texttt{temp}\,\text{K})$, and $\beta_2=E_a$.

- Coefficients of the regression model are highly sensitive to whether the interaction term is included in the model or not (because of the nonrectangular design with highly unbalanced allocation).
- Data provide no evidence of interaction.
- Strong evidence for an important voltage effect on life.

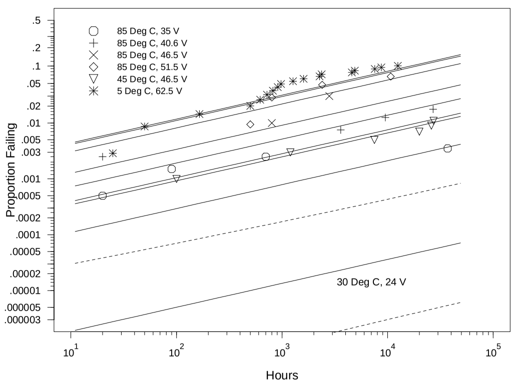
Tantalum Capacitor ALT Weibull-Inverse Power Relationship Regression ML Estimation Results

	Parameter	ML Estimate	Standard Error	95% Approximate Confidence Interval Lower	Upper
Model 1	β_0	84.4	13.6	57.8	111.
	β_1	−20.1	4.4	−28.8	−11.4
	β_2	.33	.19	−.04	.69
	σ	2.33	.36	1.72	3.16
Model 2	β_0	-78.6	109.0	−292.3	135.1
	β_1	19.9	26.7	−32.5	72.35
	β_2	5.13	3.3	−1.35	11.6
	β_3	−1.17	.80	−2.8	.40
	σ	2.33	.36	1.72	3.16

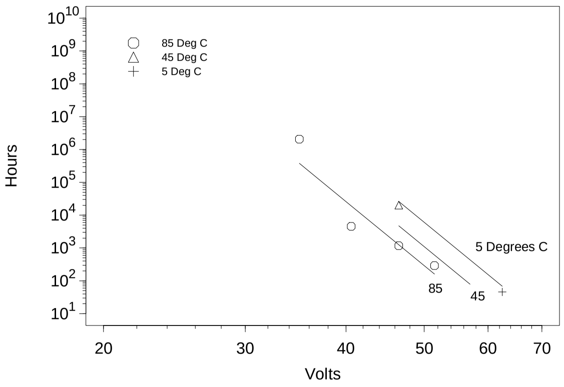
Loglikelihoods $\mathcal{L}_1=-539.63$ and $\mathcal{L}_2=-538.40$

Weibull Multiple Probability Plot of the Tantalum Capacitor ALT Data Arrhenius-Inverse Power Relationship Weibull Model (with no Interaction)

$$\widehat{\Pr}\left[T(\texttt{temp})\le t\right]=\Phi_{\text{sev}}\left[\frac{\log(t)-\widehat{\mu}(\boldsymbol{x})}{\widehat{\sigma}}\right],\ \widehat{\mu}(\boldsymbol{x})=\widehat{\beta}_0+\widehat{\beta}_1x_1+\widehat{\beta}_2x_2$$



ML Estimates of $t_{0.01}$ for the Tantalum Capacitor Life Using the Arrhenius-Inverse Power Relationship Weibull Model



Other Topics in Chapter 19

Discussion of

- Highly accelerated life tests (HALT).
- Environmental stress and STRIFE **testing**.
- Burn-in.
- Environmental stress **screening**.