Tooth Paste Market

During the 1950s, Colgate was the toothpaste market leader with about 40% of the market.

Proctor and Gamble invested heavily in the development of a new "therapeutic" toothpaste and in 1954 introduced their new brand Crest that used a fluoride-containing compound. In August 1960, they were able to convince the American Dental Association to make an endorsement saying that Crest was "an effective anti-caries" toothpaste.

In the next six years, Crest maintained a reasonably steady 15% of the market.

Over this period, Crest maintained a reasonably steady 15% of the market.

Crest's market share increased dramatically and within two years, they were the market leader. Crest's market share increased from 15% to about 50% and Proctor and Gamble conducted extensive experiments to compare Crest with toothpaste that did not contain the fluoride-containing compound.

A new "therapeutic", toothpaste and Crest introduced their new and Camile were mainly in the development of a new "therapeutic", toothpaste and Crest introduced their new and Camile were mainly in the development of Crest's market share increased from 15% to about 50% and Proctor and Gamble conducted extensive experiments to compare Crest with toothpaste that did not contain the fluoride-containing compound.

A new "therapeutic", toothpaste and Crest introduced their new and Camile were mainly in the development of Crest's market share increased from 15% to about 50% and Proctor and Gamble conducted extensive experiments to compare Crest with toothpaste that did not contain the fluoride-containing compound.

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Comparison of Models for the Crest Market Share Data

<table>
<thead>
<tr>
<th>Model</th>
<th>Noise ARIMA((p, d, q))</th>
<th>Intervention Term</th>
<th>φ₁</th>
<th>θ₁</th>
<th>β₁</th>
<th>θ₀</th>
<th>AIC</th>
<th>AICc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,1,0)</td>
<td>No</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>2</td>
<td>(1,0,1)</td>
<td>No</td>
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<td>0.664</td>
<td>0.629</td>
<td>0.368</td>
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<td>—</td>
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<tr>
<td>3</td>
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<td>0.971</td>
<td>0.658</td>
<td>0.607</td>
<td>0.370</td>
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<tr>
<td>4</td>
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<td>0.372</td>
<td>0.346</td>
<td>0.217</td>
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- **Model 1**: ARMA(1,1) (Model 1)
- **Model 2**: ARMA(1,1) (Model 2)
- **Model 3**: ARMA(1,1) with intervention term (Model 3)
- **Model 4**: ARMA(1,1) with intervention term (Model 4)

Where:

\[
I_t = \begin{cases} 
0, & \text{before August 1960,} \\
1, & \text{after August 1960,} 
\end{cases}
\]

\[
\frac{\theta_1 (B^d - 1)}{(1 - \phi B)} + \frac{\theta_0}{(1 - \phi B)} = \eta_t
\]

Models fit to the Crest Market Share Data.
Comparison of Models for the Crest Market Share Data

<table>
<thead>
<tr>
<th>Model</th>
<th>Noise</th>
<th>ARIMA(p, d, q)</th>
<th>Intervention Term</th>
<th>$\phi_1$</th>
<th>$\theta_1$</th>
<th>$\beta_1$</th>
<th>$\theta_0$</th>
<th>$\sigma^2$</th>
<th>AICc</th>
<th>$-2 \log(L_i)$</th>
<th>Ljung-Box $p$-value</th>
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<tr>
<td>1</td>
<td>Yes</td>
<td>(1,1,0)</td>
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<td>-0.992</td>
<td>-0.658</td>
<td>0.151</td>
<td>0.269</td>
<td>0.046</td>
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<td>-917.9</td>
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<td>2</td>
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<td>0.179</td>
<td>-0.162</td>
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<td>0.197</td>
<td>-0.151</td>
<td>0.269</td>
<td>0.044</td>
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<td>4</td>
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<td>Yes</td>
<td>-0.971</td>
<td>-0.197</td>
<td>0.179</td>
<td>-0.162</td>
<td>0.044</td>
<td>-933</td>
<td>-943.5</td>
<td>3.86</td>
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</table>

where

\[
I_t = \begin{cases} 
0, & \text{before August 1960} \\
1, & \text{after August 1960} 
\end{cases}
\]

Models fit to the Crest Market Share Data

Models 3 and 4 with Intervention Term (Model 3)
Two economists (Coen and Gomme) and one well-known statistician (Kendall) published a paper in a leading statistics journal, *Journal of the Royal Statistical Society*, claiming to have found a predictive model for the Financial Times Index.

The ability to use leading indicators to predict such a financial index goes against economic theory (the efficient market hypothesis).

The quarterly Financial Times Index data started with Q2 1954 and ran to the end of 1966 (51 quarters).

The OLS regression model for Financial Times Index with the Financial Times Commodity Index (lagged 7 quarters) and UK automobile production (lagged 6 quarters) as explanatory variables indicated extremely strong statistical significance.

The OLS regression model is:

\[
\text{FTindex} = \beta_0 + \beta_1 \text{FTCindex} + \beta_2 \text{UKCarProd} + \epsilon
\]

OLS assumes independent residuals. Residuals can be examined using a normal plot.

**Regression Output**

- **Dependent variable**: FTindex
- **Coefficients**:
  - (Intercept): 653.15969, p-value: 1.82e-15
  - FTCindex: -6.12736, p-value: 3.70e-13
  - UKCarProd: 0.47468, p-value: < 2e-16
- **Standard error**: 22.29 on 48 degrees of freedom
- **Multiple R-squared**: 0.9018
- **Adjusted R-squared**: 0.8977

**Residuals versus Time**

The residuals versus time plot shows no apparent pattern, suggesting that the residuals are white noise.

**Residuals versus Fitted Values**

The residuals versus fitted values plot also shows no apparent pattern, further supporting the assumption of white noise residuals.

**Residual ACF**

The autocorrelation function (ACF) plot shows no significant autocorrelation, indicating that the residuals are not autocorrelated.

**Model Specification**

The model specification used in the regression analysis is:

\[
\text{ARIMA}(0,0,0) \text{ on } \text{Index-FTindexXmat, FTindexXmatXnew}\]
Regression Relating Financial Times Index and Proposed Leading Indicators

White Noise Model for the Residuals

Model 6 esti Tabular Output

ARIMA(0,0,0)

ARIMA estimation results:

AICc: 466.3
S: 21.63

Parameter Estimation Results

<table>
<thead>
<tr>
<th>MLE</th>
<th>se</th>
<th>t.ratio</th>
<th>95% lower</th>
<th>95% upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
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<td>54.8445</td>
<td>11.9</td>
<td>545.665</td>
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<tr>
<td>FTCindex</td>
<td>-6.127</td>
<td>0.6014</td>
<td>-10.2</td>
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<tr>
<td>UKCarProd</td>
<td>0.475</td>
<td>0.0326</td>
<td>14.5</td>
<td>0.411</td>
</tr>
</tbody>
</table>

1955 1960 1965 1970

150 200 250 300 350 400 450

Actual Values, Fitted Values and Predictions with 95% Prediction Intervals

Financial Times Index

ARIMA(0,0,0) on w= Index  regr variables= rbind(FTindexXmat[, c(2, 3)], FTindexXmatXnew[, c(2, 3)])

Models Fit to the Financial Times Index Data

- Model 6: Regression with ARIMA(0,0,0) (independent) errors
  \[ y_t = \beta_0 + \beta_1 FTCindex_{t-7} + \beta_2 UKCarProd_{t-6} + \epsilon_t \]
- Model 8: Regression with ARIMA(0,1,0) (random walk) errors
  \[ y_t = \beta_1 FTCindex_{t-7} + \beta_2 UKCarProd_{t-6} + \epsilon_t \]
  \[ \epsilon_t \sim (1 - B) \]
- Model 4: ARIMA(0,1,0) (random walk) errors
  \[ y_t = \epsilon_t \]
- Model 2: IMA(1,1) errors
  \[ y_t = (1 - \theta_1 B) \epsilon_t = y_{t-1} - \theta_1 \epsilon_{t-1} + \epsilon_t \]

Comparison of Models for the Financial Times Index

<table>
<thead>
<tr>
<th>Model Number</th>
<th>6</th>
<th>8</th>
<th>4</th>
<th>2</th>
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<tbody>
<tr>
<td>( \theta )</td>
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<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>653.2</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>( \beta_1 )</td>
<td>-6.136</td>
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<tr>
<td>( \beta_2 )</td>
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<td>0.183</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( \text{Sig.} )</td>
<td>—</td>
<td>7.190</td>
<td>0.324</td>
<td>7.190</td>
</tr>
<tr>
<td>( \hat{a}_t )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>( S )</td>
<td>21.63</td>
<td>17.3</td>
<td>18.33</td>
<td>18.13</td>
</tr>
<tr>
<td>( \text{AICc} )</td>
<td>466.3</td>
<td>433.0</td>
<td>434.8</td>
<td>435.7</td>
</tr>
<tr>
<td>( -2 \log(\text{Likelihood}) )</td>
<td>458.3</td>
<td>427.0</td>
<td>432.8</td>
<td>431.7</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>19.8</td>
<td>4.37</td>
<td>1.08</td>
<td>1.46</td>
</tr>
<tr>
<td>( p )-value</td>
<td>0.003</td>
<td>0.63</td>
<td>0.98</td>
<td>0.96</td>
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</table>

Residuals versus Time

Residuals versus Fitted Values

Residual ACF

Models Fit to the Financial Times Index Data

- Model 6: Regression with ARIMA(0,0,0) (independent) errors
- Model 8: Regression with ARIMA(0,1,0) (random walk) errors
- Model 4: ARIMA(0,1,0) (random walk) errors
- Model 2: IMA(1,1) errors

Parameter Estimation Results

<table>
<thead>
<tr>
<th>MLE</th>
<th>se</th>
<th>t.ratio</th>
<th>95% lower</th>
<th>95% upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>FTCindex</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>UKCarProd</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

ARIMA(0,0,0)
Models Fit to the Financial Times Index Data

- Model 6: Regression with ARIMA(0,0,0) (independent) errors
  \[ y_t = 653.2 - 6.13 \text{FTCindex}_t - 6 + 0.47 \text{UKCarProd}_t - 6 + a_t \]

- Model 8: Regression with ARIMA(0,1,0) (random walk) errors
  \[ y_t = -1.36 \text{FTCindex}_t - 7 + 0.18 \text{UKCarProd}_t - 6 + a_t \]

- Model 4: ARIMA(0,1,0) (random walk) errors
  \[ y_t = a_t (1 - B) \]

- Model 2: IMA(1,1) errors
  \[ y_t = (1 + 0.15B) (1 - B) a_t = y_{t-1} + 0.15 a_{t-1} + a_t \]

Comparison of Models for the Financial Times Index

<table>
<thead>
<tr>
<th>Model Number</th>
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<th>(0,1)</th>
<th>(0,1,0)</th>
<th>(1,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
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<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>653.2</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-6.13</td>
<td>-1.36</td>
<td>-0.47</td>
<td>—</td>
</tr>
<tr>
<td>( \hat{a}_t )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>2.16</td>
<td>4.37</td>
<td>4.68</td>
<td>—</td>
</tr>
<tr>
<td>AICc</td>
<td>466.3</td>
<td>433.0</td>
<td>434.8</td>
<td>435.7</td>
</tr>
<tr>
<td>-2 log(Likelihood)</td>
<td>458.3</td>
<td>427.0</td>
<td>432.8</td>
<td>431.7</td>
</tr>
<tr>
<td>Ljung-Box ( \chi^2 )</td>
<td>19.8</td>
<td>4.37</td>
<td>1.08</td>
<td>1.46</td>
</tr>
<tr>
<td>Ljung-Box ( \chi^2 ) ( p)-value</td>
<td>0.003</td>
<td>0.63</td>
<td>0.98</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Module 12
Segment 3
Using a Transfer Function/Intervention Model to Explain Bad Debt Collections

Bad Debt Collections Varies

- Leading indicator: the amount of outstanding debt on the last day of the month
- Intervention variables:
  - Anticipation: \( I_1^t = \{ 1, \text{in December 1974}; 0, \text{otherwise} \) for the amount of bad debt collected in month \( t \)
  - Change: \( I_2^t = \{ 0, \text{before January 1975}; 1, \text{after January 1975} \) for the amount of bad debt collected in month \( t \)

Bad Debt Collections and Leading Indicator

Time Series BadDebtCollected

- Mean: 46.604651
- Std: 21.096831
- N: 43
- Zero Mean ADF: -1.0591
- Single Mean ADF: -0.741381
- Trend ADF: -2.876609

Input Time Series Panel
Input Series: OutstandingBadDebt

- Mean: 56.903226
- Std: 20.432982
- N: 31
- Zero Mean ADF: -1.173664
- Single Mean ADF: -0.588564
- Trend ADF: -2.167528

- Model 1: ARIMA(0,1,0) (random walk) errors
  \[ y_t = \frac{y_{t-1} - (1 - B)}{1 - B} = \frac{y_t}{1 - B} \]

- Model 2: Regression with ARIMA(0,1,0) (random walk) errors
  \[ y_t = 1.36 \text{FTCindex}_t - 7 + 0.183 \text{UKCarProd}_t - 6 + a_t \]

- Model 3: Regression with ARIMA(0,0,0) (independent) errors
  \[ y_t = 653.2 - 6.13 \text{FTCindex}_t - 7 + 0.47 \text{UKCarProd}_t - 6 + a_t \]

Models Fit to the Financial Times Index Data
Bad Debt Collections Intervention Variables

Input Series: Anticipation

<table>
<thead>
<tr>
<th>Time</th>
<th>Mean</th>
<th>Std</th>
<th>N</th>
<th>Zero Mean ADF</th>
<th>Single Mean ADF</th>
<th>Trend ADF</th>
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<tr>
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</table>

Input Series: Change

<table>
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<th>Single Mean ADF</th>
<th>Trend ADF</th>
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<tr>
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IMA(1,1) Model Estimates

Model: IMA(1, 1) No Intercept

Model Summary

<table>
<thead>
<tr>
<th>DF</th>
<th>Sum of Squared Errors</th>
<th>Variance Estimate</th>
<th>Standard Deviation</th>
<th>Akaike’s ‘A’ Information Criterion</th>
<th>Schwarz’s Bayesian Criterion</th>
<th>RSquare</th>
<th>RSquare Adj</th>
<th>MAPE</th>
<th>MAE</th>
<th>-2LogLikelihood</th>
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<tbody>
<tr>
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<td>2538.21973</td>
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<td>9.35546997</td>
<td>220.327619</td>
<td>221.728816</td>
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<td>0.80027235</td>
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<td>6.48774981</td>
<td>218.327619</td>
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Parameter Estimates

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<th>MA1</th>
<th>Lag</th>
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Forecast

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<td>1974</td>
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<tr>
<td>1974</td>
<td>1975</td>
</tr>
<tr>
<td>1975</td>
<td>1976</td>
</tr>
<tr>
<td>1976</td>
<td>1977</td>
</tr>
<tr>
<td>1977</td>
<td>1978</td>
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Prewhitening Output

Transfer Function Model Estimates

Model Summary

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<th>Sum of Squared Errors</th>
<th>Variance Estimate</th>
<th>Standard Deviation</th>
<th>Akaike’s ‘A’ Information Criterion</th>
<th>Schwarz’s Bayesian Criterion</th>
<th>RSquare</th>
<th>RSquare Adj</th>
<th>MAPE</th>
<th>MAE</th>
<th>-2LogLikelihood</th>
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<tbody>
<tr>
<td>27</td>
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Parameter Estimates

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<td>MA1,1</td>
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Transfer Function Model Residuals

Residual Value

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<td>1974</td>
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<td>-0.1353</td>
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<tr>
<td>1978</td>
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</table>

Transfer Function and Intervention Model Estimates

Model Summary

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<tr>
<th>DF</th>
<th>Sum of Squared Errors</th>
<th>Variance Estimate</th>
<th>Standard Deviation</th>
<th>Akaike’s ‘A’ Information Criterion</th>
<th>Schwarz’s Bayesian Criterion</th>
<th>RSquare</th>
<th>RSquare Adj</th>
<th>MAPE</th>
<th>MAE</th>
<th>-2LogLikelihood</th>
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<td>25</td>
<td>349.967178</td>
<td>13.9986785</td>
<td>3.74148079</td>
<td>163.431976</td>
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<td>0.84688442</td>
<td>4.7284338</td>
<td>2.21010938</td>
<td>155.431976</td>
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</table>

Parameter Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>OutstandingBadDebt</th>
<th>Anticipation</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>Scale</td>
<td>Scale</td>
<td>Scale</td>
</tr>
<tr>
<td></td>
<td>MA1,1</td>
<td>MA1,1</td>
<td>MA1,1</td>
</tr>
<tr>
<td>Factor</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lag</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>Estimate</td>
<td>0.72381</td>
<td>3.05494</td>
<td>-20.38407</td>
</tr>
</tbody>
</table>

Prewhitening Output
Bad Debt Collections

Transfer Function and Intervention Model Residuals

-15

-10

-5

0

5

Residual Value

1973.0
1973.5
1974.0
1974.5
1975.0
1975.5
1976.0

Time

Models Fit to the Bad Debt Collections Data

• Model 1: IMA(1,1)
  \( y_t = (1 - \theta B) a_t \)
  \( y_t = (1 - \theta B)(1 - B) a_t \)

• Model 2: IMA(1,1) with leading indicator
  \( y_t = \omega_0 x_t (1 - B) + (1 - \theta B) a_t \)
  \( y_t = \omega_0 x_{t-1} + (1 - \theta B)(1 - B) a_t \)

• Model 3: IMA(1,1) with leading indicator and intervention terms
  \( y_t = \omega_0 x_t (1 - B) I_{1t} + \omega_2 I_{2t} + (1 - \theta B) a_t \)
  \( y_t = \omega_0 x_{t-1} + \omega_1 I_{1t} + \omega_2 I_{2t} + (1 - \theta B)(1 - B) a_t \)

Estimated Model Parameters for the Bad Debt Collections Data

• Model 1: IMA(1,1) univariate
  \( y_t = (1 - 0.22 B)(1 - B) a_t \)

• Model 2: IMA(1,1) with leading indicator
  \( y_t = 0.95 x_t - 1 + (1 - 0.42 B)(1 - B) a_t \)

• Model 3: IMA(1,1) with leading indicator and intervention terms
  \( y_t = 0.72 x_t - 1 + 3.05 I_{1t} - 20.38 I_{2t} + (1 - 0.77 B)(1 - B) a_t \)

Comparison of Models for Bad Debt Collections

<table>
<thead>
<tr>
<th>Model Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMA(1,1)</td>
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<td></td>
<td></td>
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<tr>
<td>Transfer</td>
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<td></td>
</tr>
<tr>
<td>Univariate</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ljung-Box \( \chi^2 \) p-value

- Model 1: IMA(1,1) with leading indicator and intervention
  \( \chi^2 = 6.99, p = 0.82 \)

- Model 2: IMA(1,1) with leading indicator
  \( \chi^2 = 9.77, p = 0.53 \)

- Model 3: IMA(1,1) with leading indicator and intervention terms
  \( \chi^2 = 15.74, p = 0.36 \)

Multivariate (Vector) Time Series Models

VARIMA Models

Vector Time Series Definition

At each time period we observe a vector of \( m \) observations.

Examples:

- Daily common stock closing price of Apple, Facebook, Google, Twitter, and Microsoft.
- Monthly total sales and advertising expenses for a company.
- Monthly total sales and advertising expenses for a company.

There are many ways to express Vector Time Series models.

The approach given here follows Practical Experiences with Modeling and Forecasting Time Series by G.M. Jenkins.

Module 12
Segment 4

\( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \)

\( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \)

\( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \) + \( \phi(b - 1) \) + \( \theta(b - 1) \)

Models fit to the Bad Debt Collections data.
Vector ARIMA Model is a generalization of the ARIMA model to multiple time series. It is defined as:

\[ (\varphi(B)) \left[ (\theta(B))^T \right] Z_t = \left[ \epsilon_t \right] \]

where \( \varphi(B) \) and \( \theta(B) \) are matrices of polynomials in the lag operator \( B \), \( Z_t \) is the \( m \times 1 \) vector of time series at time \( t \), and \( \epsilon_t \) is the \( m \times 1 \) vector of residuals at time \( t \).

The complete realization of the time series \( Z_t \) is given by:

\[ Z_t = \left[ Z_1 \ldots Z_m \right] = \left[ \epsilon_1 \ldots \epsilon_m \right] \]

where \( \epsilon_t \) is the \( m \times 1 \) vector of residuals at time \( t \).
Fitted Vector ARIMA Model

SL\[t = SL[t - 1] + 0.73(SL[t - 1] - MP[t - 1] - 0.17(MP[t - 1] - 0.61aSL[t - 12] + aSL[t])

NF\[t = NF[t - 12] + 0.31(NF[t - 12] - MP[t - 12] - 0.51(MP[t - 12] - 0.55(MP[t - 2] - 0.45(CP[t - 13] - 1) - 0.4aMP[t - 12] + aNF[t - 12] - 0.77aMP[t - 13]) - 0.11aCP[t - 12] + aMP[t])

Meat Price and Crop Price

Note feedback relationship between
• Meat Price and New Feed
• Meat Price and Crop Price

Comparison of Multivariate and Univariate Model Predictions for NewFeed

1981 New Feed Predictions

Comparison of Multivariate and Univariate Model Predictions for Slaughter

1981 Slaughter Predictions

Comparison of Multivariate and Univariate Model Predictions for NewFeed

1981 New Feed Predictions

Comparison of Multivariate and Univariate Model Predictions for Slaughter

1981 Slaughter Predictions

Vector AR (VAR) Model

VAR model

VAR model parameters may be harder to interpret.

VAR models tend to not be parsimonious.

Special case of VARIMA with only AR terms in the model

• VAR models tend to not be parsimonious.
• VAR model parameters may be harder to interpret.

• With a sufficient amount of data, it is easier to identify a
  VAR model.
• More software is available for VAR modeling.

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• Meat Price and New Feed
• Meat Price and Crop Price

Note feedback relationship between
• Meat Price and New Feed
• Meat Price and Crop Price

Field Fitted Vector ARIMA Model

For Cattle Economic Variables
Software for Fitting VARIMA and VAR Models

• VARIMA Models
  ◦ R package `fArma` (in CRAN)
  ◦ SCA (www.scausa.com)

• VAR Models
  ◦ R package `vars` (in CRAN)
  ◦ SAS Econometrics and Time Series (ETS) (www.sas.com)
  ◦ STATA (www.stata.com)

All of these packages also support univariate and transfer function ARIMA models.

Software for Fitting VARIMA and VAR Models

and other Time Series Models