Two steps-ahead (\( \hat{Z} \))

Forecast Error and Forecast Error Variances

Four quantities inside \([ \_ \_ \_ \_ ]\), substitute value if known, forecast \( \hat{Z} \)

For quantities inside \([ \_ \_ \_ \_ ]\), substitute value if known, forecast \( \hat{Z} \)

In general, steps-ahead:

\[
(\hat{Z} + \hat{Z} + 1)_{n} = \text{Var}(1)_{n} \hat{Z} \]

\[
1 + \hat{Z} + \hat{Z} + 1 + \hat{Z} + 1 = (1)_{n} \hat{Z} - \hat{Z} + \hat{Z} \]

\[
\cdots + \hat{Z} + \hat{Z} + 1 + \hat{Z} + 1 + \cdots + \hat{Z} + \hat{Z} + \hat{Z} = (1)_{n} \hat{Z} \]

\[
\hat{Z} + \hat{Z} + 1 + \hat{Z} + \hat{Z} + \hat{Z} = (1)_{n} \hat{Z} \]

\[
\hat{Z} + 1 + \hat{Z} + 1 + \hat{Z} + 1 + \cdots + \hat{Z} + \hat{Z} + \hat{Z} = (1)_{n} \hat{Z} \]

One step-ahead (\( \hat{Z} \))

The ARIMA model for \( Z \)

Forecast Errors and Forecast Error Variances

\[
\text{Var}(1)_{n} \hat{Z} = \sigma^{2} \]

\[
\hat{Z} = \theta_{0} \]

\[
\phi_{0} = \hat{Z} \]

\[
\psi_{0} = \hat{Z} \]

\[
\cdots \]

\[
\theta_{0} + \phi_{0} + \psi_{0} = \hat{Z} \]

\[
\hat{Z} = \theta_{0} \]

\[
\phi_{0} = \hat{Z} \]

\[
\psi_{0} = \hat{Z} \]

\[
\cdots \]

\[
\theta_{0} + \phi_{0} + \psi_{0} = \hat{Z} \]

The ARMAs for \( Z \) are

\[
\text{ARIMA Model Forecast Equation in Infinite MA Form} \]

\[
\text{ARIMA Model Forecast Equation in Infinite MA Form} \]

\[
\text{ARIMA Model Forecast Equation in Infinite MA Form} \]

\[
\text{ARIMA Model Forecast Equation in Infinite MA Form} \]

Forecasting Multiple Values From an ARIMA Model

Forecast Errors and Forecast Error Variances

Forecast Errors and Forecast Error Variances

Forecast Errors and Forecast Error Variances

Forecast Errors and Forecast Error Variances

Forecast Errors and Forecast Error Variances

Forecast Errors and Forecast Error Variances

Forecast Errors and Forecast Error Variances

Forecast Errors and Forecast Error Variances
AR Models and Forecasts for the Sunspot Process

AR Models—Part 1

The Wolfer Sunspot Numbers 1770-1869
Function esti

Output based on the Square Roots of the Wolfer Sunspot Numbers 1770-1869
AR(1) Model—Part 1

Residuals vs. Time
Time
Residuals
1780 1800 1820 1840 1860
-2 0 2 4

Wolfer Sunspots
Model: Component 1 :: ar: 1 on w = (Number of Spots)^0.5

Residual ACF
Lag
ACF
0 10 20 30
-1.0 0.0 0.5 1.0

Residuals vs. Fitted Values

Fitted Values
Residuals
2 4 6 8 10
-2 0 2 4

Normal Probability Plot
Residuals
Normal Scores
-2 -1 0 1 2
-2 -1 0 1 2

Index
Number of Spots
1780 1800 1820 1840 1860 1880
0 50 100 150

Wolfer Sunspots
1770-1869
Function esti

Output based on the Square Roots of the Wolfer Sunspot Numbers 1770-1869
AR(1) Model—Part 2

Actual Values * * *  Fitted Values * * * Future Values
AR(2) Model—Part 1

Function and Output for the Wolfer Sunspot Numbers 1770-1869

AR(2) Model—Part 2

Function and Output for the Wolfer Sunspot Numbers 1770-1869

ARIMA(0,1,0) Model

Comparison of Models for the Support Data
Plot of AR(2) Model Log-likelihood Surface for the Wolfer Sunspot Numbers 1770-1869

Output based on the Square Roots of Residuals

AR(3) Model—Part 1

Sunspot Process and Eventual Forecasts
Higher-Order AR Models and Forecasts for the Segment 3

Module 8

Square Root Wolfer Sunspot Data
Plot of AR(2) Model Log-likelihood Surface for the Wolfer Sunspot Numbers 1770-1869

Normal Probability Plot

AR(3) Model—Part 2
For $q_0, \cdots, q_q$ and $p_0, \cdots, p_p$ for $\phi_1, \cdots, \phi_q$ and $\psi_1, \cdots, \psi_p$ we substitute estimates for parameters, giving approximate prediction intervals (for $e_t$). Substituting $\hat{e}_t$ for $e_t$, and so on, when computing prediction intervals from data.

A 95% prediction interval for $Z$ is $\hat{Z} \pm 1.96 \sqrt{\text{var}(\hat{Z})}$. For three steps-ahead it simplifies to

$$\hat{Z} + 1.96 \sqrt{\hat{Z}^2}$$

For two steps-ahead the simpler to

$$\hat{Z} + 1.96 \hat{Z}$$

For one step-ahead the simpler to

$$\hat{Z} + 1.96 \sqrt{\text{var}(\hat{Z})}$$

A 95% prediction interval for $Z^t$ steps-ahead is

$$\hat{Z} + 1.96 \sqrt{\text{var}(\hat{Z})}$$

ARIMA($p,d,q$) Model—Part 1

The output square number 1770-1869

Function $q_0$ could be based on the mean roots of $\phi_1, \cdots, \phi_q$.
For nonstationary time series, things are more complicated, but the forecast-error variance grows without bound because the weights do not sum to one. For stationary time series, from the equations above, we can see that because

\[ \lim_{n \to \infty} \sum_{i=-n}^{n} a_i = 1 \]

the forecast-error variance grows without bound. For nonstationary time series, things are more complicated.

\[ (\varphi(z))^{-1} = (\cdots + \psi_3 + \psi_2 + 1)z^{-1} \]

In the end, we have

\[ \varphi(z) = (1 - \varphi_1 z^{-1} - \cdots - \varphi_p z^{-p}) \]

because the weights die down, the long-run forecast is

\[ \cdots + \psi_1 z^{-2} + \psi_0 z^{-1} + \varphi_1 z^{-1} + \cdots + 0 + 0 = \lim_{n \to \infty} (\theta_n)z \]

For stationary time series, from

Eventual (long-run) Forecasts
With fewer parameters, forecasts less sensitive to deviations.

Model may be applied more generally to similar processes.

Reasons for Needing a Long Realization

- Can check model stability by dividing data in two and analyzing.
- Possible to check forecasts by withholding recent data.
- Fewer estimation problems (likelihood function behaves better).
- Are known (good approximation of realization is large).
- Approximate prediction intervals assume the parameters are known (good approximation of realization is large).
- Estimate seasonal pattern (need at least 4 or 5 seasonal periods).
- Estimate correlation structure (i.e., the ACF and PACF).

Comparison of Models for the Savings Rate Data

<table>
<thead>
<tr>
<th>Model</th>
<th>p</th>
<th>d</th>
<th>q</th>
<th>AIC</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>224.5</td>
<td>0.32</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>224.0</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>218.4</td>
<td>0.70</td>
</tr>
</tbody>
</table>

ARIMA(0,1,1) Model

Trend

Some Practical Considerations and Deterministic Segment 5