Statistics 451 | Final Examination | Name ________________________

Spring 1999 | **You must show all of your work**

1. After using the differencing scheme

$$W_t = (1 - \mathrm{B}^4)Z_t$$

it was determined that an appropriate model for W_t would be $W_t = (1 - \theta 1\mathrm{B})(1 - \Theta_1\mathrm{B}^4)a_t$.

(a) Write down the "unscrambled" model equation for Z_t (i.e., a model equation depending on a finite number of past values of Z and a).

$Z_t =$

(b) Is the model for Z_t stationary or not or do we not have enough information to decide? Explain.

(c) Derive an expression for ρ_5, the the autocorrelation for W_t at lag 5.

(d) What is the *practical* interpretation of ρ_5?

2. Derive expressions for the ψ weights ψ_1, ψ_2, ψ_3 for an ARMA(1,1) model.

3. A time series Z_t can be described by an ARMA(1,1) model. Assume that a realization $Z_1, Z_2, \ldots Z_{100}$ is available to estimate all of the parameters in this model and to compute forecasts and that a forecast is needed for Z_{103}, using Z_{100} as the forecast origin.

   (a) Give an expression for Z_{103}, based on the ARMA(1,1) model.

   (b) Give an expression that can be used to *compute* $\widehat{Z}_{100}(3)$, the forecast for Z_{103}.

   (c) Give an expression for $e_{100}(3)$, the forecast error in $\widehat{Z}_{100}(3)$.

   (d) Give an expression for the variance of $e_{100}(3)$.

   (e) Give an expression for a 95% prediction interval for Z_{103}.

4. Derive $\text{Cov}(Z_t, a_{t-2})$ for an ARMA(1,1) model.

5. The range mean plot provides a preliminary indication of the need for a transformation. There are, however, better tools to help make the final decision on what transformation (if any) to use. What are these tools are how are they used?

6. Consider the following transfer function model that was suggested to relate the demand for furniture to housing starts in a county in Southern California, using monthly data.

$$
\begin{aligned}
Y_t &= \nu(\mathrm{B})X_t + n_t \\
&= \frac{\omega_3 B^3}{(1-\delta_1 \mathrm{B})} X_t + \frac{1-\theta_1 \mathrm{B}}{(1-\mathrm{B})} a_t, \quad a_t \sim \mathrm{nid}(0, \sigma_a^2)
\end{aligned}
$$

(a) Find the unscrambled equation giving Y_t as a function of only the parameters and a finite number of lagged values of Y_t, X_t, and a_t.

(b) Briefly explain (and draw a graph to illustrate) the behavior of the transfer function

$$\nu(\mathrm{B}) = \frac{\omega_3 B^3}{(1-\delta_1 \mathrm{B})}$$

to a pulse (or impulse) input function if $\delta_1 = 0.10$ and $\omega_3 = 2$.

(c) Without doing any derivations, briefly explain the effect that having to predict future values of the input X_t will have on the prediction standard error. Why is this important?

7. Describe the role and importance of sensitivity analysis in time series forecasting.