Statistics 451 | Final Examination | Name ________________________

Spring 1998 | **You must show all of your work**

1. Experience has shown us that doing a transformation on the response variable can have an important effect on the resulting forecasts.

   (a) List some of the things relating to data or a model that are affected by a transformation.

   (b) The range mean plot provides a preliminary indication of the need for a transformation. There are, however, better tools to help make the final decision on what transformation (if any) to use. Explain.

2. Assume that the variability in a stationary time series process can be described by a normal distribution. As the realization size become large, the width of a confidence interval for the mean of the process shrinks to 0. What happens to the width of the corresponding prediction interval to capture the next observation? Why? Be as specific as possible.

3. What is the practical interpretation of ϕ_1 in an AR(1) time series model? That is, how would you explain it to someone who knows a little about statistics, but nothing about time series analysis?

4. What information or data do you need in order to compute a "true" ACFs and PACFs? (i.e., what would be the input to a computer program used to compute these functions?)

5. The Yule-Walker equations play an important role in time series theory and methods.

   (a) Write down the Yule-Walker equations for the AR(2) model.

   (b) Explain (i.e., list the steps) how the Yule-Walker equations can be used to find the true PACF for an MA(2) model.

6. A sample realization can be described by the model

$$Z_t = \theta_0 - \theta_1 a_{t-1} + a_t, \quad a_t \sim \text{nid}(0, \sigma_a^2).$$

   An analyst decides to try the differencing scheme

$$W_t = (1 - \mathrm{B})Z_t.$$

   (a) Write down the model equation for the derived time series W_t. Note that Z needs to be eliminated from the model.

   (b) Is W_t invertible? Why or why not?

   (c) Give expressions for the mean and variance of Z_t.

   (d) Give expressions for the mean and variance of W_t. Comment.

7. Consider the following seasonal time series model

$$W_t = (1 - \Theta_1 \mathrm{B}^4)a_t, \quad a_t \sim \mathrm{nid}(0, \sigma_a^2)$$

with the differencing scheme

$$W_t = (1 - \mathrm{B}^4)Z_t$$

where Z_t is measured in thousands of dollars.

(a) Write down the unscrambled equation for Z_t.

(b) Derive $\mathrm{Var}(W_t)$.

(c) Derive ρ_4 and ρ_8 for W_t.

8. An important application of Box-Jenkins methods is to generate forecasts for explanatory variables being used in regression models and to substitute these forecasts for the unknown explanatory variables in order to generate forecasts for the regression response variable. For example, suppose that the number of people coming to eat dinner at my restaurant today is related to the number of advertising spots run by my primary competitor today. If I want to forecast the number of people coming to eat dinner at my restaurant tomorrow, I would need to use a forecast for the number of spots that my competitor will run tomorrow. Explain the implications that this has for the computation of predictions and prediction intervals.

9. Consider the following intervention models for amount of deposits for a bank. These models have been suggested to describe the effect of a change in ownership of a bank.

$$\text{Model 1:} \quad Z_t = \omega_0 I_t + \frac{1-\theta_1 \mathrm{B}}{(1-\mathrm{B})} a_t, \quad a_t \sim \mathrm{nid}(0, \sigma_a^2)$$

$$\text{Model 2:} \quad Z_t = \frac{\omega_0}{(1-\mathrm{B})} I_t + \frac{1-\theta_1 \mathrm{B}}{(1-\mathrm{B})} a_t, \quad a_t \sim \mathrm{nid}(0, \sigma_a^2)$$

where I_t is a step function changing from 0 to 1 at the time of the announcement of the change.

(a) For both models, find the unscrambled equation giving Z_t as a function of only the parameters and a finite number of lagged values of Z_t, I_t and a_t.

(b) Briefly explain (and draw a graph to illustrate) the behavior of these two transfer function models. Which model seems more plausible? Justify your answer.