Statistics 451 | Final Examination | Name ________________________
Spring 1997 | | **You must show all of your work**

1. Experience has shown us that doing a transformation on the response variable can have an important effect on the resulting forecasts. The range mean plot provides a preliminary indication of the need for a transformation. There are, however, better tools to help make the final decision on what transformation (if any) to use. Explain.

2. Assume that the variability in a stationary time series process can be described by a normal distribution. As the realization size become large, the width of a confidence interval for the mean of the distribution shrinks to 0. What happens to the width of the corresponding prediction interval to capture the next observation? Why? Be as specific as possible.

3. A sample realization can be described by the model

$$Z_t = \theta_0 + \phi_1 Z_{t-1} + a_t.$$

An analyst decides to try the differencing scheme

$$W_t = (1 - \mathrm{B})^2 Z_t$$

(a) Write down the model equation for the derived time series W_t. Note that Z needs to be eliminated from the model.

(b) Is W_t stationary? Why or why not?

(c) Is W_t invertible? Why or why not?

4. What is the practical interpretation of ϕ_1 in an AR(1) time series model? That is, how would you explain it to someone who knows a little about statistics, but nothing about time series analysis?

5. Consider the nonseasonal time series model

$$Z_t = \theta_0 + (1 - \theta_1 \text{B}) a_t$$

   (a) Derive the expression for the true ACF values ρ_1 and ρ_2 for this model.

   (b) Give expressions for the true PACF values ϕ_{11} and ϕ_{22} for this model.

   (c) Explain the role that the Yule-Walker equations play in part 5b.

6. What information or data do you need in order to compute a "sample" ACFs and PACFs? (i.e., what would be the input to a computer program used to compute these functions?)

7. Consider the seasonal time series model for sales of the Xwave company, defined by

$$W_t = \theta_0 + (1 - \Theta_1 \mathrm{B}^{12})a_t$$

with the differencing scheme

$$W_t = (1 - \mathrm{B}^{12})Z_t$$

where Z_t is measured in thousands of dollars.

(a) Write down the unscrambled equation for Z_t.

(b) What is the practical interpretation of θ_0 in this model (be precise, including the units of θ_0 in your answer)?

(c) Write down the unscrambled equation for W_t (note, that Z_t should not be included in this unscrambled equation).

(d) Derive $\mathrm{Var}(W_t)$.

(e) Derive ρ_{12} and ρ_{13} for W_t.

8. The sales of Company A (y) depends on the number of radio advertising spots used by Company B (x) in the recent past. Daily time series data are available for both variables. An analyst has identified the transfer function model as

$$y_t = \nu_0 x_t + \nu_1 x_{t-1} + (1 - \phi_1 \mathrm{B})^{-1} a_t \tag{1}$$

The model for x has been identified as an AR(2).

(a) Write down the unscrambled model for y (only finite lags allowed).

(b) Draw a simple picture to illustrate the inputs and the outputs of the transfer function model.

(c) Explain how one could identify the form of the transfer function model by using the cross correlation function.

(d) Explain how one could find simple estimates of ν_0, ν_1, ν_2, and ν_3 by using the cross correlation function.

(e) It is common practice to use an ARMA model to generate forecasts for inputs to a dynamic regression model like the one in (1). An analyst first used Splus to fit AR(2) model for x and generate forecasts for x up to six steps ahead. Then the transfer function model was fit with Splus. The forecasts for x were used with the transfer function model to generate forecasts and prediction intervals six steps ahead for y. Explain why the prediction intervals could be misleading.