Statistics 451 Examination 2 Name _____
Spring 1999 **You must show all of your work**

When asked to explain something, provide an explanation that could be understood by someone who does not have formal training in statistical methods.

1. A time series Z_t can be described by an AR(1) model. Assume that a realization $Z_1, Z_2, \ldots Z_{100}$ is available to estimate all of the parameters in this model and to compute forecasts and that a forecast is needed for Z_{102}, using Z_{150} as the forecast origin.

   (a) Give an expression for Z_{102}, based on the AR(1) model.

   (b) Give an expression that can be used to compute $\widehat{Z}_{100}(2)$, the forecast for Z_{102}.

   (c) Give an expression for $e_{100}(2)$, the forecast error in $\widehat{Z}_{100}(2)$.

   (d) Give an expression for the variance of $e_{100}(2)$.

   (e) Give an expression for a 95% prediction interval for Z_{102}.

2. Increasing one's sample size will provide more precision for estimation. Increased precision is reflected, for example, by narrower confidence intervals. In the limit, with a very large sample size, the width of a confidence interval will approach 0. For a prediction interval, the width of the interval is large sample sizes will *not* approach 0. Provide an intuitive explanation for why it is that confidence intervals shrink to 0 but prediction intervals do not, as one's sample (or realization) size increases.

3. The AR(1) model can be expressed by

$$Z_t = \theta_0 + \phi_1 Z_{t-1} + a_t, \quad a_t \sim \text{nid}(0, \sigma_a^2).$$

(a) Show how to express this model as an infinite MA model.

(b) Derive the Yule-Walker equation for this model.

4. After using the differencing scheme

$$W_t = (1 - \text{B}) Z_t$$

it was determined that an appropriate model for W_t would be an MA(1).

(a) Write down the "unscrambled" model equation for Z_t (i.e., a model equation depending on a finite number of past values of Z and a).

$Z_t =$

(b) Is the model for Z_t stationary or not? Explain.

(c) Is the model for W_t stationary or not? Explain.

5. Briefly explain how the range-mean plot can be useful in time series modeling. What is its proper place in the modeling strategy?

6. Briefly explain why the sample ACF of the residuals is an important diagnostic for time series modeling.

7. You have been asked to develop a model that can be used for forecasting weekly sales of a company, based on data from 95 past weeks. Forecasts are needed for up to 7 future weeks. Suppose that sales are not seasonal. After your analysis you end up with two different models. One model is stationary and the other is nonstationary. From looking at all of the commonly suggested model diagnostics, both models pass all tests. Explain briefly how you develop the needed forecasts.

8. A pure seasonal moving average model of order 1 (i.e., $Q = 1$) for monthly data is

$$W_t = (1 - \Theta_1 \mathrm{B}^{12})a_t$$

Derive the true autocorrelation function value ρ_{12} for this model.