Statistics 451 Examination 2 Name ________________________

Spring 1997 **You must show all of your work**

1. Briefly explain why and how sample ACF and PACF's differ from "true" ACF and PACF's.

2. Briefly explain the idea behind "maximum likelihood" estimation.

3. The AIC model-choice criterion uses the statistic

$$\text{AIC} = -2\log(\text{Likelihood}) + 2m$$

where m is the number of estimated parameters in the model.

   (a) What is the rational for $2m$ term in this criterion?

   (b) Why is it *not* a good idea to just automatically choose the model with the smallest value of AIC?

4. The range-mean plot is a useful tool, during the tentative identification stage, to suggest whether a transformation might be needed or not. After fitting a particular model, however, a plot of the residuals versus the fitted values (one-step-ahead forecasts) provides a clearer picture of whether the chosen transformation was appropriate or not. We should, however, put more weight on the information in the residual plot rather than the range-mean plot.

   (a) Why should the plot of the residuals versus the fitted values carry more weight in our modeling decision making?

   (b) If the residual plot is to take precedence, why bother at all with the range-mean plot?

5. The Yule-Walker equations for an AR(3) model (in correlation form) can be expressed as:

$$\begin{aligned}\rho_1 &= \phi_1\rho_0 + \phi_2\rho_1 + \phi_3\rho_2 \\ \rho_2 &= \phi_1\rho_1 + \phi_2\rho_0 + \phi_3\rho_1 \\ \rho_3 &= \phi_1\rho_2 + \phi_2\rho_1 + \phi_3\rho_0\end{aligned}$$

where $\rho_0 = 1$. Explain conceptually (list steps, but details and formulas are not needed) how these equations can be used to

(a) Obtain simple non-iterative estimates of the AR(3) model.

(b) Obtain an expression for a true PACF value (say ϕ_{33}) for an MA model.

6. Consider the AR(1) model

$$Z_t = \theta_0 + \phi_1 Z_{t-1} + a_t.$$

(a) Show how to express this model as an infinite MA model.

(b) Derive the mean of this model.

(c) Derive the Yule-Walker equation for this model.

7. Explain why $\mathrm{Cov}(Z_{t-1}, a_t) = 0$ for any ARMA model.

8. Consider the following ARMA models. For each, determine if the model is stationary or not.

(a)

$$(1 - .7\mathrm{B})(1 - \mathrm{B})Z_t = (1 - .3B)a_t$$

(b)

$$(1 - 1.6\mathrm{B} + .64\mathrm{B}^2)Z_t = (1 - 1.5B)a_t$$

9. A time series Z_t can be described by an MA(1) model. Assume that a realization $Z_1, Z_2, \ldots Z_{150}$ is available to estimate all of the parameters in this model and compute forecasts and that a forecast is needed for Z_{154}, using Z_{150} as the forecast origin.

(a) Give an expression for Z_{154}, based on the MA(1) model.

(b) Give an expression that can be used to compute $\widehat{Z}_{150}(4)$, the forecast for Z_{154}.

(c) Give an expression for $e_{150}(4)$, the forecast error in $\widehat{Z}_{150}(4)$.

10. Briefly explain the difference between a confidence interval and a prediction interval.

11. A sample realization can be described by the model

$$Z_t = \phi_1 Z_{t-1} + a_t.$$

An analyst decides to try the differencing scheme

$$W_t = (1 - \mathrm{B})^2 Z_t$$

(a) Write down the model equation for W_t.

(b) Is W_t stationary? Why or why not?

(c) Is W_t invertible? Why or why not?