Statistics 451 Examination 1 Name ________________________

Spring 1997 **You must show all of your work**

When asked to explain something, provide an explanation that could be understood by someone who does not have formal training in statistical methods.

1. A regression model such as

$$Y_t = \beta_0 + \beta_1 \text{Time} + \sum_{i=2}^{12} \beta_j w_j + a_t, \quad a_t \sim \text{nid}(0, \sigma_a^2) \qquad (1)$$

can be useful for describing a process generating monthly time series data. In this model, $w_2 = 1$ in February and 0 otherwise, $w_3 = 1$ in March and 0 otherwise, ..., $w_{12} = 1$ in December and 0 otherwise and Time is a coded time variable. One concern with the use of such a model is, however, that the residuals may not really be independent, but may exhibit some autocorrelation.

(a) *List* and briefly describe useful methods for determining whether the residuals are autocorrelated or not.

(b) Briefly describe the possible consequences of unrecognized autocorrelation on the point estimate of a time series slope.

(c) Briefly describe the possible consequences of unrecognized autocorrelation when judging the strength of statistical evidence (significance) of a regression coefficient.

(d) Briefly describe the possible consequences of unrecognized autocorrelation when computing a forecast for a future observation.

(e) If Time is defined as Time $= 1, 2, \ldots, n$ for $n = 144$ observations (12 observations each year for 12 years), what is the interpretation of β_1 in this model?

(f) The coefficients $\beta_2, \ldots, \beta_{12}$ are used to estimate seasonal effects in the model. Some of these may be positive and some may be negative. Expalin what this implies.

2. For purposes of description and modeling it is sometimes useful to consider the different "components" of a time series model. Briefly explain the difference between the periodic (also known as seasonal) component and the cyclical components of a time series. Give an example of each.

3. Suppose that the following autoregression model can be used to adequately describe a time series.

$$Z_t = \theta_0 + \phi_1 Z_{t-1} + a_t, \quad a_t \sim \text{nid}(0, \sigma_a^2) \tag{2}$$

Suppose also that $\phi_1 = .8$.

(a) What is the interpretation of ϕ_1 in this model?

(b) What is the interpretation of σ_a^2 in this model?

(c) Derive $\sigma_Z^2 \equiv \text{Var}(Z_t)$ for this model.

(d) What is the interpretation of σ_Z^2 in this model. How does it relate to σ_a^2?

4. Briefly explain how simulation can be used to help one interpret a normal probability plot.

5. Briefly explain the interpretation of the *sample* partial autocorrelation function and how it can be used to help identify an ARMA model.

6. It is important to distinguish between the "true" or "theoretical" ACF and PACF and the "sample" ACF and PACF. Both are important in the consideration of time series modeling and analysis.

   (a) What is the difference between the "true" or "theoretical" ACF and the "sample" ACF?

   (b) What information or inputs does one need to compute, numerically, a "true" or "theoretical" ACF?

   (c) What information or inputs does one need to compute, numerically, a "sample" ACF?

7. Briefly explain why a realization of size 300 would be better than a realization of size 75 when trying to identify an ARMA model.

8. Consider the following MA time series model

$$Z_t = 10 + (1 - .3\mathrm{B})(1 - \delta\mathrm{B})a_t, \quad a_t \sim \mathrm{nid}(0, \sigma_a^2)$$

   where $\sigma_a^2 = 4$ and δ is unspecified.

   (a) Find the unscrambled form of the model for Z_t.

   (b) Find the roots or expressions for the roots of the defining MA polynomial (hint: there is a very simple method for doing this).

   (c) Under what conditions (i.e., for what values of δ) will this model be invertable?