Module 9

Seasonal ARIMA (SARIMA) Models

Class notes for Statistics 451: Applied Time Series
Iowa State University

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21h 15min
Module 9

Segment 1

Seasonal Time Series Examples and the SARIMA Model
Seasonal Time Series

- “Periodic” is a better term; “seasonal” commonly used
- Seasonal (periodic) model with $S$ observations per period
  - Monthly data has 12 observations per year
  - Quarterly data has 4 observations per year
  - Daily data has 5 or 7 (or some other number) of observations per week.
- Notes:
  - Sunspot data is cyclical, not seasonal, because distance between peaks is random.
  - Just because we have 12 observations per year, does not mean that there is seasonal behavior (e.g., stock prices show no regular seasonal patterns)
Energy Consumption Data

Residential and Commercial Energy Consumption 1982-1993

![Graph showing energy consumption data from 1982 to 1993. The graph indicates fluctuations in energy consumption with a general upward trend over the years.](image-url)
Machine Tool Shipments

Machine-Tool Shipments 1968-1975
US Gas and Oil Consumption

US Consumption of Gas and Oil 1967-1982

<table>
<thead>
<tr>
<th>Year</th>
<th>Billions of Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>20</td>
</tr>
<tr>
<td>1969</td>
<td>40</td>
</tr>
<tr>
<td>1971</td>
<td>60</td>
</tr>
<tr>
<td>1973</td>
<td>80</td>
</tr>
<tr>
<td>1975</td>
<td>90</td>
</tr>
<tr>
<td>1977</td>
<td>110</td>
</tr>
<tr>
<td>1979</td>
<td>130</td>
</tr>
<tr>
<td>1981</td>
<td>150</td>
</tr>
<tr>
<td>1983</td>
<td>170</td>
</tr>
</tbody>
</table>
Des Moines Precipitation

Time

Inches


0 2 4 6 8 10
Seasonal Differencing

- Seasonal differencing is usually needed. For example,

\[ W_t = (1 - B^{12})Z_t = Z_t - B^{12}Z_t = Z_t - Z_{t-12} \]
\[ Z_t = Z_{t-12} + W_t \]

or

\[ W_t = (1 - B)(1 - B^{12})Z_t = Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13} \]
\[ Z_t = Z_{t-1} + Z_{t-12} - Z_{t-13} + W_t \]

- More generally, the “working series” is

\[ W_t = (1 - B)^d(1 - B^S)^DZ_t \]

implemented, for example by `idens(airline.tsd, gamma=0, d=1, D=1)`

Even more generally (not supported by RTSERIES)

\[ W_t = (1 - B)^d(1 - B^{S_1})^{D_1}(1 - B^{S_2})^{D_2}Z_t \]

and so on.
Seasonal ARIMA Model (SARIMA)

The SARIMA($p, d, q \times (P, D, Q)_S$) model is

$$
\Phi_P(B^S)\phi_p(B)(1 - B)^d(1 - B^S)^DZ_t = \Theta_Q(B^S)\theta_q(B)a_t \\
\Phi_P(B^S)\phi_p(B)W_t = \Theta_Q(B^S)\theta_q(B)a_t
$$

For example using $P = 1, p = 1, Q = 2, q = 1, S = 12$ gives

$$
\Phi_1(B^{12}) = (1 - \Phi_1 B^{12}) \\
\phi_1(B) = (1 - \phi_1 B) \\
\Theta_2(B^{12}) = (1 - \Theta_1 B^{12} - \Theta_2 B^{24}) \\
\theta_1(B) = (1 - \theta_1 B)
$$

The modeling problem is to choose a transformation ($\gamma$ and $m$), differencing ($d$ and $D$), and the SARIMA model ($p, q, P, Q$) for the working series $W_t$. As before, use tsplot, range-mean plot, ACF and PACF.

By multiplying out the differencing terms, the SARIMA model can be expressed in the ARMA form:

$$
\phi_p^*(B)Z_t = \theta_q^*(B)a_t.
$$
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Segment 2

Introduction to Seasonal Time Series Modeling
Revisiting the Airline Data
Data Analysis Strategy

1. Tentative Identification
   - Time Series Plot
   - Range-Mean Plot
   - ACF and PACF

2. Estimation
   - Least Squares or
   - Maximum Likelihood

3. Diagnostic Checking
   - Residual Analysis and Forecasts

4. Model ok?
   - No
   - Yes

   - Forecasting
   - Explanation
   - Control

   Use the Model
Strategy for Seasonal Time Series Modeling

- Plot data
- Use a range-mean plot to see if a transformation might be needed; choose tentative $\gamma$ value (and perhaps $m > 0$).
- Choose differencing scheme(s).
  - Look at $3 \times S + 3$ lags on the ACF and PACF of $W_t$ for all combinations of $d = 0, 1$ and $D = 0, 1$.
  - Go higher with $d$ or $D$ as needed.
  - Choose the stationary $W_t$ with the smallest $d$ and $D$.
    Avoid over-differencing
- Tentatively identify a model(s) from the ACF and PACF of the chosen differencing scheme(s) [i.e., choose $(p, d, q)(P, D, Q)$].
- Fit, check, and compare models.
- Iterate as necessary
- At the end, choose alternative value of $\gamma$, if needed
Airline Data identification Output Log Transformation
No Differencing
esti(airline.tsd, gamma=0)

International Airline Passengers
w = log(Thousands of Passengers)

Range-Mean Plot

ACF

PACF
Airline Data iden Output Log Transformation
One Regular Difference
esti(airline.tsd, gamma=0, d=1)

International Airline Passengers
\( w = (1-B)^1 \log(\text{Thousands of Passengers}) \)

ACF

PACF
Airline Data i d e n Output Log Transformation
One Seasonal Difference
esti(airline.tsd, gamma=0, D=1)

International Airline Passengers
\( w = (1-B^{12})^1 \log(\text{Thousands of Passengers}) \)

ACF

PACF
Airline Data identification Output Log Transformation
Regular and Seasonal Difference
\texttt{esti(airline.tsd, gamma=0, d=1, D=1)}

International Airline Passengers
\( w = (1-B)^{12} \log( \text{Thousands of Passengers} ) \)

ACF

PACF
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Segment 3

True ACF and PACF for Seasonal Time Series
SARIMA Models
Methods of Obtaining the True ACF and PACF for Seasonal Models

- Derivations similar to nonseasonal models. Use the un-scrambled form of the model for $W_t$.

- Formula sheets from Box and Jenkins give the ACF. Note the use of special cases (e.g., setting $\theta = 0$ in model 1 gives the ACF of a particularly simple seasonal model $W_t = -\Theta_1 a_{t-S} + a_t$)

- PACF computed using the same Yule-Walker based “Durbin formula” as in the nonseasonal models

- `plot.true.acfpacf()` RTSERIES function. For example

  ```r
  plot.true.acfpacf(model=list(ma=c(0.5,0,0,0,0, 0,0,0,0,0,0,0.9,-0.45)),nacf=38) or
  ```

  ```r
  plot.true.acfpacf(model=list(ma=c(0.5,rep(0,10),0.9,-0.45)),nacf=38)
  ```
**Example:** \( W_t = (1 - \Theta_1 B^{12})a_t = -\Theta_1 a_{t-12} + a_t \)

This is a special case of Model 1 from the Box and Jenkins formula sheet with \( \theta_1 = 0 \). Everything is easy to derive.

\[
\begin{align*}
\gamma_0 &= E(W_t^2) = (1 + \Theta_1^2)\sigma_a^2 \\
\gamma_1 &= \gamma_2 = \cdots = \gamma_{11} = 0 \\
\gamma_{12} &= E(W_t W_{t+12}) = E[(-\Theta_1 a_{t-12} + a_t)(-\Theta_1 a_t + a_{t+12})] \\
&= 0 + \cdots + 0 + E(-\Theta_1 a_t^2) = -\Theta_1 \sigma_a^2 \\
\gamma_{13} &= \gamma_{14} = \cdots = 0
\end{align*}
\]

\[
\rho_j = \frac{\gamma_j}{\gamma_0}
\]
\[
\rho_1 = \rho_2 = \cdots = \rho_{11} = 0
\]
\[
\rho_{12} = \frac{\gamma_{12}}{\gamma_0} = -\frac{\Theta_1}{(1 + \Theta_1^2)}
\]
\[
\rho_j = 0, \quad j > 12.
\]
True ACF/PACF for $W_t = -\Theta_1 a_{t-12} + a_t$, $\Theta_1 = 0.9$

```
plot.true.acfpacf(model=list(ma=c(rep(0, 11), +0.9)), nacf=38)
```
True ACF/PACF for $W_t = -\Theta_1 a_{t-12} + a_t$, $\Theta_1 = -0.9$

```r
plot.true.acfpacf(model=list(ma=c(rep(0, 11), -0.9)), nacf=38)
```

**True ACF**

```
ma: 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.9
```

**True PACF**

```
ma: 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.9
```
**Example:** \((1 - \Phi_1 B^{12})W_t = a_t, \quad W_t = \Phi_1 W_{t-12} + a_t\)

This is a special case of Model 2 from the Box and Jenkins formula sheet with \(\theta_1 = \Theta_1 = 0\).

\[
\gamma_0 = \mathbb{E}(W_t^2) = \frac{\sigma_a^2}{1 - \Phi_1^2}, \quad -1 < \Phi_1 < 1
\]

\[
\gamma_{12} = \mathbb{E}(W_t W_{t+12}) = \Phi_1 \gamma_0
\]

\[
\gamma_1 = \gamma_2 = \cdots = \gamma_{11} = 0
\]

\[
\gamma_j = \Phi_1 \gamma_{j-12}, \quad j > 12.
\]

\[
\rho_j = \gamma_j / \gamma_0
\]

\[
\rho_1 = \rho_2 = \cdots = \rho_{11} = 0
\]

\[
\rho_{12} = \gamma_{12} / \gamma_0 = \Phi_1
\]

\[
\rho_j = \Phi_1 \rho_{j-12}, \quad j > 12.
\]
True ACF and PACF for \( W_t = \Phi_1 W_{t-12} + a_t \) with
\[ \Phi_1 = 0.9 \]

\[
\text{plot.true.acfpacf(model=list(ar=c(rep(0, 11), +0.9)), nacf=38)}
\]
True ACF and PACF for \( W_t = +\Phi_1 W_{t-12} + a_t \) with \( \Phi_1 = -0.9 \)

plot.true.acfpacf(model=list(ar=c(rep(0, 11), -0.9)), nacf=38)

**True ACF**

```
ar: 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.9
```

**True PACF**

```
ar: 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.9
```
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Segment 4

True ACF and PACF for the Airline Model
Example: \( W_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12})a_t \)

This is Model 1 from the Box and Jenkins formula sheet.

\[
W_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12})a_t \\
= (1 - \theta_1 B - \Theta_1 B^{12} + \theta_1 \Theta_1 B^{13})a_t \\
= -\theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1 \Theta_1 a_{t-13} + a_t
\]

This is like an MA(13) and, again, everything is easy to derive. For example,

\[
\gamma_{11} = E(W_t W_{t-11}) \\
= 0 + 0 + \cdots + E[(\Theta_1 a_{t-12})(-\theta_1 a_{t-12})] + 0 + \cdots \\
= \theta_1 \Theta_1 \sigma_a^2
\]

\[
\gamma_0 = \\
\gamma_1 = \\
\gamma_{12} = \\
\gamma_{13} =
\]
Coefficients for the Multiplicative Model
Expressed as an MA(13)

\[ W_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12})a_t \]
\[ = (1 - \theta_1 B - \Theta_1 B^{12} + \theta_1 \Theta_1 B^{13})a_t \]
\[ = -\theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1 \Theta_1 a_{t-13} + a_t \]
\[ = -\theta_1 a_{t-1} - \theta_1 \Theta_1 a_{t-12} - \theta_1 \Theta_1 a_{t-13} + a_t \]

The coefficients for this model are:

\[ \theta_1 = \theta_1 = 0.50 \]
\[ \theta_{12} = \Theta_1 = 0.90 \]
\[ \theta_{13} = -\theta_1 \times \Theta_1 = -0.45 \]
True ACF/PACF for $W_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12})a_t$, 
$\theta_1 = 0.5, \Theta_1 = 0.9$

plot.true.acfpacf(model=list(ma=c(0.5,rep(0,10),0.9,-0.45)),nacf=38)
**True ACF/PACF for** \( W_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12})a_t, \)
\( \theta_1 = 0.1, \Theta_1 = 0.9 \)

plot.true.acfpacf(model=list(ma=c(0.1,rep(0,10),0.9,-0.09)),nacf=38)
True ACF/PACF for $W_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12}) a_t$,

$\theta_1 = -0.5$, $\Theta_1 = -0.9$

plot.true.acfpacf(model=list(ma=c(-0.5,rep(0,10),-0.9,-0.45)),nacf=38)
True ACF/PACF for $(1 - \phi_1 B)(1 - \Phi_1 B^{12})W_t = a_t,$

$\phi_1 = 0.5, \Phi_1 = 0.9$

plot.true.acfpacf(model=list(ar=c(+0.5,rep(0,10),+0.9,-0.45)),nacf=38)
True ACF/PACF for \((1 - \phi_1 B)(1 - \Phi_1 B^{12})W_t = a_t,\)

\[\phi_1 = 0.1, \Phi_1 = 0.9\]

plot.true.acfpacf(model=list(ar=c(+0.1,rep(0,10),+0.9,-0.09)),nacf=38)
True ACF/PACF for \((1 - \phi_1 B)(1 - \Phi_1 B^{12})W_t = a_t\),

\(\phi_1 = -0.5, \Phi_1 = -0.9\)

plot.true.acfpacf(model=list(ar=c(-0.5,rep(0,10),-0.9,-0.45)),nacf=38)
True ACF/PACF for $(1 - \Phi_1 B^{12})W_t = (1 - \Theta_1 B^{12})a_t$,

$\Phi_1 = -0.9, \Theta_1 = 0.9$

model=list(ar=c(rep(0,11), -0.9), ma=c(rep(0, 11), +0.9))
True ACF/PACF for \((1 - \Phi_1 B^{12})W_t = (1 - \Theta_1 B^{12})a_t\),

\(\Phi_1 = 0.9, \Theta_1 = -0.9\)

model=list(ar=c(rep(0,11), +0.9), ma=c(rep(0, 11), -0.9))
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Segment 5

Comparison of Models for the Airline Data Using the Log Transformation
Airline Passenger Model Summary Table

<table>
<thead>
<tr>
<th>Statistics 451</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERNATIONAL AIRLINE PASSENGER EXAMPLE MODEL COMPARISON</td>
</tr>
<tr>
<td>Log transformation</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Box-Cox Transform Parameter Y</td>
</tr>
<tr>
<td>d = 0</td>
</tr>
<tr>
<td>D = 0</td>
</tr>
<tr>
<td>( \theta_1 )</td>
</tr>
<tr>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>( \theta_3 )</td>
</tr>
<tr>
<td>( \theta_4 ) (Seasonal)</td>
</tr>
<tr>
<td>Significant ( \theta_4(m) )</td>
</tr>
<tr>
<td>s = ( \theta_4 )</td>
</tr>
<tr>
<td>AIC</td>
</tr>
<tr>
<td>-2(Log Likelihood)</td>
</tr>
<tr>
<td>L-B ( \chi^2 ) (20 dof)</td>
</tr>
<tr>
<td>p-value for L-B</td>
</tr>
<tr>
<td>Number of parameters</td>
</tr>
</tbody>
</table>

Notes: 1) Values in parentheses are t-ratios or p-values
2) AIC = -2(Log Likelihood) + 2M where M is the total number of parameters estimated, which helps correct for lack of parsimony.
3) It is not possible to compare s, -2(Loglikelihood) or AIC for different values of the same log parameter model.
Airline Data esti AR(1)
Log Transformation (Model 1) Output Part 1
esti(airline.tsd, gamma=0, model=model.pdq(p=1),
  y.range=c(100, 1100))
Airline Data esti AR(1)
Log Transformation (Model 1) Output Part 2

International Airline Passengers
Model: Component 1 :: ar: 1 on w= log(Thousands of Passengers )
Actual Values *** Fitted Values *** Future Values
** 95 % Prediction Interval **

Normal Probability Plot
The “Airline Model”

Differencing $d = 1$ and $D = 1$ to give the working series

$$W_t = (1 - B)(1 - B^{12})Z_t$$
$$= Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}$$

Model for the working series $W_t$

$$W_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12})a_t$$
$$= -\theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1 \Theta_1 a_{t-13} + a_t$$

The unscrambled multiplicative “airline model” for $Z_t$ is

$$Z_t = Z_{t-1} + Z_{t-12} - Z_{t-13} - \theta_1 a_{t-1} - \Theta_1 a_{t-12} + \theta_1 \Theta_1 a_{t-13} + a_t$$

and has only 2 parameters. The unscrambled additive “airline model” for $Z_t$ is

$$Z_t = Z_{t-1} + Z_{t-12} - Z_{t-13} - \theta_1 a_{t-1} - \theta_12 a_{t-12} - \theta_13 a_{t-13} + a_t$$

has three parameters.
Airline Data

SARIMA(0, 1, 0)(0, 1, 1)_12

Log Transformation (Model 3) Output Part 1

`esti(airline.tsd, gamma=0, model=model.pdq(d=1, D=1, Q=1), y.range=c(100, 1100))`

International Airline Passengers

Model: Component 1 :: ndiff= 1  Component 2 :: period= 12 ndiff= 1 ma: 1 on w= log( Thousands of Passengers )

Residuals vs. Time

Residual ACF

Fitted Values

Residuals vs. Fitted Values
Airline Data esti SARIMA(0,1,1)(0,1,1)_12
Log Transformation (Model 4) Output Part 1
esti(airline.tsd, gamma=0, model=model.pdq(d=1, q=1, D=1, Q=1), y.range=c(100, 1100))

International Airline Passengers
Model: Component 1 :: ndiff=1 ma:1 Component 2 :: period=12 ndiff=1 ma:1 on w=log(Thousands of Passengers )
**Airline Data** estimated SARIMA$(0, 1, 1)(0, 1, 1)_12$

Log Transformation (Model 4) Output Part 2
Airline Data Estimated AR(1)
Log Transformation (Model 1) Output Part 2

International Airline Passengers
Model: Component 1 :: ar: 1 on w= log(Thousands of Passengers )
Actual Values *** Fitted Values *** Future Values
*** 95 % Prediction Interval ***

Normal Probability Plot

Residuals
Normal Scores
-2 -1 0 1 2
Index
Thousands of Passengers
Airline Data Additive Airline Model
Log Transformation (Model 5) Output Part 1
esti(airline.tsd, gamma=0, model=add.airline.model,
y.range=c(100,1100))

International Airline Passengers
Model: Component 1 :: ndiff= 1 ma: 1 12 13  Component 2 :: period= 12 ndiff= 1 on w= log(Thousands of Passengers )
Residuals vs. Time

Residuals vs. Fitted Values
Residual ACF
Airline Data Additive Airline Model

Log Transformation (Model 5) Output Part 2

International Airline Passengers
Model: Component 1 :: ndiff= 1 ma: 1 12 13 Component 2 :: period= 12 ndiff= 1 on w= log(Thousands of Passengers )

Actual Values *** Fitted Values *** Future Values
** 95 % Prediction Interval **

![Graph showing International Airline Passengers data](image)

Normal Probability Plot

![Normal Probability Plot](image)
Airline Data

**Airline Data**

SARIMA(0, 1, 1)(0, 1, 1)\(_{12}\)

Log Transformation (Model 4) Output Part 2

International Airline Passengers

Model: Component 1 :: ndiff= 1 ma: 1 Component 2 :: period= 12 ndiff= 1 ma: 1 on w= log(Thousands of Passengers )

Actual Values *** Fitted Values *** Future Values

** 95 % Prediction Interval **

Normal Probability Plot

Normal Scores

Index

Thousands of Passengers

Index

Normal Scores

Residuals
## Airline Passenger Model Summary Table

<table>
<thead>
<tr>
<th>Statistics 451</th>
<th>INTERNATIONAL AIRLINE PASSENGER EXAMPLE MODEL COMPARISON</th>
<th>Log transformation</th>
<th>Strong</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Box-Cox Transform Parameter Y</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Regular differences</td>
<td>d = 0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Seasonal differences</td>
<td>D = 0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>.96</td>
<td>(43.1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>.39</td>
<td>(4.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>.40</td>
<td>(.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>.39</td>
<td>(5.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>.44</td>
<td>(5.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>.30</td>
<td>(3.7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$ (Seasonal)</td>
<td>.60</td>
<td>(8.6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$ (Seasonal)</td>
<td>.56</td>
<td>(7.7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$ (Seasonal)</td>
<td>.58</td>
<td>(8.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significant $\theta_1$</td>
<td>.11</td>
<td>(1.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significant $\theta_1$ (log)</td>
<td>1.4, 8, 11</td>
<td>3.12, 23</td>
<td>1.23</td>
<td>23</td>
</tr>
<tr>
<td>Significant $\theta_1$ (log)</td>
<td>12, 20, 24, 36</td>
<td>23 (23 close)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s = $\theta_1$</td>
<td>.104</td>
<td>.043</td>
<td>.039</td>
<td>.036</td>
</tr>
<tr>
<td>AIC</td>
<td>-234.87</td>
<td>-451.98</td>
<td>-469.55</td>
<td>-485.39</td>
</tr>
<tr>
<td>-2(Log Likelihood)</td>
<td>-238.87</td>
<td>-453.98</td>
<td>-471.55</td>
<td>-489.39</td>
</tr>
<tr>
<td>L-B $\chi^2$ (20 dfs)</td>
<td>209.1</td>
<td>44.9</td>
<td>33.2</td>
<td>15.9</td>
</tr>
<tr>
<td>p-value for L-B</td>
<td>(0)</td>
<td>(.001)</td>
<td>(.03)</td>
<td>(.72)</td>
</tr>
<tr>
<td>Number of parameters</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

**Notes:**
1) Values in parentheses are t-ratios or p-values
2) AIC = -2(Log Likelihood) + 2M where M is the total number of parameters estimated, which helps correct for lack of parsimony.
3) It is not possible to compare s, -2(Loglikelihood) or AIC for different values of the Box-Cox parameter Y.
Module 9

Segment 6

Comparison of Models for the Airline Data Using Alternative Transformations
### Airline Passenger Model Summary Table

<table>
<thead>
<tr>
<th>Statistics 451</th>
<th>INTERNATIONAL AIRLINE PASSENGER EXAMPLE MODEL COMPARISON</th>
<th>Log transformation</th>
<th>Strong</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Box-Cox Transform</td>
<td>Parameter ψ</td>
<td>0 0 0 0 0 -.333 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular differences</td>
<td>d = 0</td>
<td>1 1 1 1 1 1 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seasonal differences</td>
<td>D = 0</td>
<td>1 1 1 1 1 1 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| θ₁ | .96  
(43.1) | .39  
(4.3) | .40  
(5.0) | .39  
(5.0) | .44  
(5.5) | .30  
(3.7) | | |
| θ₂ | - - - - - - - | - - - - - - - |        |      |
| θ₃ | - - - - - - - | - - - - - - - |        |      |
| θ₄ (Seasonal) | - - - .60  
(8.6) | .56  
(7.7) | .58  
(8.2) | .11  
(1.2) |        |      |
| Significant θ₄(s) | 1.4,8,11  
12,20,24,36 | 3.12,23  
12,20,24,36 | 1.23  
23 | 23  
(23 close) | 23 |      |
| s = θ₄ | .106  
.043 | .039  
.036 | .036  
.002 | 11.6 | 1019 |
| AIC | -234.87  
-451.98 | -469.55  
-485.39 | -486.04  
-1221.8 | 1015 |
| -2(Log Likelihood) | -238.87  
-453.98 | -471.55  
-489.39 | -490.04  
-1225.8 | 1015 |
| L-B χ² (20DoF) | 209.1  
44.9 | 33.2  
15.9 | 19.07 | 19.07 | 24.3 |
| p-value for L-B | (0)  
(.001) | (.03)  
(.72) | (.51)  
(.23) | (.51) |  |
| Number of parameters | 2  
1 | 1  
2 | 3  
2 | 2  
2 |

**Notes:**
1) Values in parentheses are t-ratios or p-values.
2) AIC = -2(Log Likelihood) + 2M where M is the total number of parameters estimated, which helps correct for lack of parsimony.
3) It is not possible to compare s, -2(Loglikelihood) or AIC for different values of the same parameter ψ.
Airline Data esti SARIMA(0,1,1)(0,1,1)_{12}
No Transformation (Model 7) Output Part 1
esti(airline.tsd, gamma=1, model=model.pdq(d=1, q=1, D=1, Q=1), y.range=c(100, 1100))
**Airline Data**

**SARIMA(0,1,1)(0,1,1)_{12}**

No Transformation (Model 7) Output Part 2

**International Airline Passengers**

Model: Component 1: ndiff=1 ma:1 Component 2: period=12 ndiff=1 ma:1 on w= Thousands of Passengers

Actual Values *** Fitted Values *** Future Values

** 95 % Prediction Interval **

**Normal Probability Plot**

![Normal Probability Plot]
Airline Data esti SARIMA(0, 1, 1)(0, 1, 1)_{12}

Log Transformation (Model 4) Output Part 2

International Airline Passengers
Model: Component 1 :: ndiff= 1 ma: 1 Component 2 :: period= 12 ndiff= 1 ma: 1 on w= log(Thousands of Passengers )
Actual Values *** Fitted Values *** Future Values
** 95 % Prediction Interval **

Normal Probability Plot

Residuals

Normal Scores

-2 -1 0 1 2

-0.10 -0.05 0.0 0.05 0.10

Thousands of Passengers

Index


0 200 400 600 800 1000
Airline Data

SARIMA(0, 1, 1)(0, 1, 1)_{12}

\[ \gamma = -0.333 \]

Transformation (Model 6) Output Part 1

\[ \text{esti(airline.tsd, gamma=-0.333, model=model.pdq(d=1, q=1, } \]
\[ \text{D=1, Q=1), y.range=c(100, 1100))} \]
Airline Data estimated SARIMA(0, 1, 1)(0, 1, 1)_{12}

$\gamma = -0.333$ Transformation (Model 6) Output Part 2
Airline Data estimated SARIMA$(0, 1, 1)(0, 1, 1)_12$

Log Transformation (Model 4) Output Part 2
### Airline Passenger Model Summary Table

<table>
<thead>
<tr>
<th>Statistics 451</th>
<th>INTERNATIONAL AIRLINE PASSENGER EXAMPLE MODEL COMPARISON</th>
<th>Log transformation</th>
<th>Strong</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Box-Cox Transform Parameter Y</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Regular differences</td>
<td></td>
<td>d = 0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Seasonal differences</td>
<td></td>
<td>D = 0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\hat{\eta}_1$</td>
<td></td>
<td>.96</td>
<td>(43.1)</td>
<td>.39</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td></td>
<td>(4.3)</td>
<td>(5.0)</td>
<td>(?)</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_1$ (Seasonal)</td>
<td></td>
<td>-</td>
<td>-</td>
<td>.60</td>
</tr>
<tr>
<td>Significant $\theta_{s(a)}$</td>
<td></td>
<td>1.4, 8, 11</td>
<td>3.12, 23</td>
<td>1.23</td>
</tr>
<tr>
<td>s = $\theta_4$</td>
<td></td>
<td>12, 20, 24, 36</td>
<td>.043</td>
<td>.039</td>
</tr>
<tr>
<td>AIC</td>
<td></td>
<td>-234.87</td>
<td>-451.98</td>
<td>-469.55</td>
</tr>
<tr>
<td>$-2$(Log Likelihood)</td>
<td></td>
<td>-238.87</td>
<td>-453.98</td>
<td>-471.55</td>
</tr>
<tr>
<td>L-B $\chi^2$ (20 DoF)</td>
<td></td>
<td>209.1</td>
<td>44.9</td>
<td>33.2</td>
</tr>
<tr>
<td>p-value for L-B</td>
<td></td>
<td>(0)</td>
<td>(.001)</td>
<td>(.03)</td>
</tr>
<tr>
<td>Number of parameters</td>
<td></td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Notes:**

1) Values in parentheses are t-ratios or p-values

2) AIC = $-2$(Log Likelihood) + 2M where M is the total number of parameters estimated, which helps correct for lack of parsimony.

3) It is not possible to compare s, $-2$(loglikelihood) or AIC for different values of the non-linear parameter $\theta$. 

9 - 57
Module 9

Segment 7

Seasonal Differencing of Nonseasonal Data Can be Misleading
Graphical Output from Function `iden` for Simulated Series #B with 1 Regular Difference

Simulated Time Series #B
\[ w = (1 - B)^1 \text{ Sales} \]

**Graphs**

- **ACF**
- **PACF**

**Axes**
- **w** vs. **time**
- **Lag**
- **ACF** partial ACF
- **Partial ACF**
Graphical Output from Function `iden` for Simulated Series #B with 1 Regular and 1 Seasonal Difference

Simulated Time Series #B
\[ w = (1-B)^1 (1-B^{12})^1 \text{ Sales} \]

ACF

PACF
Function esti Output for Simulated Series #B
SARIMA(0, 1, 1)(0, 0, 0)_{12} Model—Part 2

Simulated Time Series #B
Model: Component 1 :: ndiff= 1 ma: 1 on w= Sales
Actual Values *** Fitted Values *** Future Values
** 95 % Prediction Interval **

Normal Probability Plot

Normal Scores

Residuals
Function Output for Simulated Series #B
SARIMA(0, 1, 1)(0, 1, 1)_{12} Model—Part 2

Simulated Time Series #B
Model: Component 1: ndiff=1 ma: 1 Component 2: period=12 ndiff=1 ma: 1 on w= Sales
Actual Values *** Fitted Values *** Future Values
*** 95 % Prediction Interval ***

Normal Probability Plot

Residuals
Normal Scores
-30 -20 -10 0 10 20
-2 -1 0 1 2
9 - 62
Seasonal Over-Differencing

Assume that

$$(1 - B)Z_t = a_t$$

so that $Z_t$ has a random walk model. Then

$$W_t = (1 - B)Z_t = a_t$$

will follow a “trivial model” and we will not expect to see anything in the ACF/PACF. What if we take seasonal differences?

$$W_t = (1 - B^{12})(1 - B)Z_t = (1 - B^{12})a_t$$

$$= -a_{12} + a_t$$

which is a noninvertible MA(12)!
Module 9

Segment 8

Ohio Power Example Model Identification
Ohio Power Consumption Data

Ohio Electric Power Consumption

![Graph showing Ohio electric power consumption from 1954 to 1970. The y-axis represents billions of kilowatt-hours, and the x-axis represents time from 1954 to 1970. The graph shows a steady increase in power consumption over the years.]
Ohio Power Consumption Data iden Output
No Transformation and No Differencing
iden(ohio.tsd, gamma=1)

Ohio Electric Power Consumption
w= Billions of Kilowatt-hours

Range-Mean Plot

ACF

PACF
Ohio Power Consumption Data iden Output
No Transformation and One Regular Difference
iden(ohio.tsd, gamma=1, d=1)

Ohio Electric Power Consumption
w = (1-B)^1 Billions of Kilowatt-hours

ACF

PACF

ACF

Partial ACF
Ohio Power Consumption Data iden Output
No Transformation and Two Regular Differences
iden(ohio.tsd, gamma=1, d=2)

Ohio Electric Power Consumption
\( w = (1-B)^2 \) Billions of Kilowatt-hours

ACF

PACF

Partial ACF

Lag
Ohio Power Consumption Data \texttt{iden} Output
No Transformation and Three Regular Differences
\texttt{iden(ohio.tsd, gamma=1, d=3)}

Ohio Electric Power Consumption
\(w = (1-B)^3\) Billions of Kilowatt-hours

\begin{figure}
\centering
\includegraphics[width=\textwidth]{ohio_power_consumption}
\caption{Ohio Electric Power Consumption}
\end{figure}

\begin{figure}
\centering
\begin{minipage}[b]{0.45\textwidth}
\textbf{ACF}
\end{minipage}\hfill
\begin{minipage}[b]{0.45\textwidth}
\textbf{PACF}
\end{minipage}
\caption{ACF and PACF plots for Ohio Electric Power Consumption}
\end{figure}
Ohio Power Consumption Data iden Output
No Transformation and One Seasonal Difference
iden(ohio.tsd, gamma=1, D=1)

Ohio Electric Power Consumption
\[ w = (1 - B^{12})^1 \text{ Billions of Kilowatt-hours} \]

ACF

PACF
Ohio Power Consumption Data iden Output
No Transformation Regular & Seasonal Difference
iden(ohio.tsd, gamma=1, d=1, D=1)

Ohio Electric Power Consumption
w= (1-B)^1 (1-B^12)^1 Billions of Kilowatt-hours

ACF

PACF
Module 9

Segment 9

Ohio Power Example Model Comparison Using No Transformation
Ohio Power Consumption Model Summary Table

<table>
<thead>
<tr>
<th>Statistics 451</th>
<th>OHIO POWER EXAMPLE MODEL COMPARISON</th>
<th>Log Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box-Cox Transformation</td>
<td>No Transformation</td>
<td>1</td>
</tr>
<tr>
<td>Parameter γ</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Regular differences</td>
<td></td>
<td>d = 0</td>
</tr>
<tr>
<td>Seasonal differences</td>
<td></td>
<td>D = 1</td>
</tr>
<tr>
<td>$\hat{\phi}_1$</td>
<td></td>
<td>.617</td>
</tr>
<tr>
<td>(8.8)</td>
<td>(10.0)</td>
<td>(8.0)</td>
</tr>
<tr>
<td>$\hat{\phi}_2$</td>
<td></td>
<td>.293</td>
</tr>
<tr>
<td>(4.2)</td>
<td>(3.7)</td>
<td>(-1.8)</td>
</tr>
<tr>
<td>$\hat{\theta}_1$</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>(5.2)</td>
<td>(4.4)</td>
<td>(4.9)</td>
</tr>
<tr>
<td>$\hat{\theta}_1$ (seasonal)</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>(9.5)</td>
<td>(11.8)</td>
<td>(9.5)</td>
</tr>
<tr>
<td>Significant $\phi_k$ (a)</td>
<td></td>
<td>2,7,8,10,12</td>
</tr>
<tr>
<td>s = $\hat{\delta}_s$</td>
<td></td>
<td>1.80</td>
</tr>
<tr>
<td>AIC</td>
<td></td>
<td>759.08</td>
</tr>
<tr>
<td>$-2(\text{Log Likelihood})$</td>
<td></td>
<td>755.08</td>
</tr>
<tr>
<td>L-B $\chi^2$ (20df)</td>
<td></td>
<td>71.37</td>
</tr>
<tr>
<td>(0)</td>
<td>(0)</td>
<td>(.01)</td>
</tr>
<tr>
<td>Numer of parameters</td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Notes: 1) Values in parentheses are t-ratios or p-values

2) $\text{AIC} = -2(\text{Log Likelihood}) + 2M$ where $M$ is the total number of parameters estimated, which helps correct for lack of parsimony.

3) It is not possible to compare s, $-2(\text{log likelihood})$ or AIC for different values of the Box-Cox parameter $\gamma$. 

9 - 73
Ohio Power Consumption Data

SARIMA(2, 0, 0)(0, 1, 0)_{12}

No Transformation (Model 1) Output Part 1

esti(ohio.tsd, gamma=1, model=model.pdq(p=2, D=1))
Ohio Power Consumption Data
SARIMA(2, 0, 0)(0, 1, 0)_{12}
No Transformation (Model 1) Output Part 2

Ohio Electric Power Consumption
Model: Component 1 :: ar: 1 2 Component 2 :: period= 12 ndiff= 1 on w= Billions of Kilowatt hours
Actual Values * * * Fitted Values * * * Future Values
* * 95 % Prediction Interval * *
Ohio Power Consumption Data
SARIMA(2, 0, 0)(0, 1, 1)$_{12}$
No Transformation (Model 2) Output Part 1
esti(ohio.tsd, gamma=1, model=model.pdq(p=2, D=1, Q=1))
Ohio Power Consumption Data
SARIMA(2, 0, 0)(0, 1, 1)_12
No Transformation (Model 2) Output Part 2

Ohio Electric Power Consumption
Model: Component 1 :: ar: 1 2 Component 2 :: period= 12 ndiff= 1 ma: 1 on = Billions of Kilowatt-hours
Actual Values *** Fitted Values *** Future Values
** 95 % Prediction Interval **

Normal Probability Plot
Normal Scores
Residuals

Billions of Kilowatt-hours
Index

1955 1960 1965 1970
20 40 60 80
Ohio Power Consumption Data

SARIMA(2, 0, 0)(0, 1, 0)_12

No Transformation (Model 1) Output Part 2

Ohio Electric Power Consumption

Model: Component 1: ar: 1 2 Component 2: period= 12 ndiff= 1 on w= Billions of Kilowatt-hours
Actual Values *** Fitted Values *** Future Values
** 95 % Prediction Interval **

Normal Probability Plot

Index

Billions of Kilowatt-hours

Normal Scores

Residuals
Ohio Power Consumption Data

SARIMA(2, 0, 1)(0, 1, 1)_12

No Transformation (Model 3) Output Part 1

esti(ohio.tsd, gamma=1, model=model.pdq(p=2,q=1,D=1,Q=1))

Ohio Electric Power Consumption

Model: Component 1 ::  ma: 1 ar: 1 2  Component 2 ::  period= 12 ndiff= 1 ma: 1 on w= Billions of Kilowatt-hours

Residuals vs. Time

Residuals vs. Fitted Values

Residual ACF
Ohio Power Consumption Data

SARIMA(2, 0, 1)(0, 1, 1)_12

No Transformation (Model 3) Output Part 2

Ohio Electric Power Consumption

Model: Component 1: ma: 1 ar: 1 2 Component 2: period=12 ndiff=1 ma: 1 on w= Billions of Kilowatt-hours

Actual Values * * * Fitted Values * * * Future Values

* * 95 % Prediction Interval * *
Ohio Power Consumption Data
SARIMA(2, 0, 0)(0, 1, 1)_{12}
No Transformation (Model 2) Output Part 2

Ohio Electric Power Consumption
Model: Component 1 :: ar: 1 2 Component 2 :: period= 12 ndiff= 1 ma: 1 on w= Billions of Kilowatt-hours
Actual Values *** Fitted Values *** Future Values
*** 95 % Prediction Interval ***

Normal Probability Plot
Residuals
Ohio Power Consumption Data
SARIMA(0,1,1)(0,1,1)_{12} (Airline)
No Transformation (Model 5) Output Part 1
esti(ohio.tsd, gamma=1, model=model.pdq(d=1, q=1, D=1, Q=1))

Ohio Electric Power Consumption
Model: Component 1 :: ndiff=1 ma:1 Component 2 :: period=12 ndiff=1 ma:1 on w= Billions of Kilowatt-hours

Residuals vs. Time

Residuals vs. Fitted Values

Residual ACF
Ohio Power Consumption Data
SARIMA(0, 1, 1)(0, 1, 1)_{12} (Airline)
No Transformation (Model 5) Output Part 2
Ohio Power Consumption Data

SARIMA(2, 0, 1)(0, 1, 1)_12

No Transformation (Model 3) Output Part 2

Ohio Electric Power Consumption

Model: Component 1: ma: 1 ar: 1 2 Component 2: period= 12 ndiff= 1 ma: 1 on w= Billions of Kilowatt-hours

Actual Values ** Fitted Values ** Future Values

** 95 % Prediction Interval **
# Ohio Power Consumption Model Summary Table

## Statistics 451

<table>
<thead>
<tr>
<th>Statistics 451</th>
<th>OHIO POWER EXAMPLE MODEL COMPARISON</th>
<th>Log Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box-Cox Transformation</td>
<td>No Transformation</td>
<td>Transformation</td>
</tr>
<tr>
<td>Parameter γ</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>d = 0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D = 1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Seasonal differences

<table>
<thead>
<tr>
<th>β₁</th>
<th>β₂</th>
<th>β₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>.617</td>
<td>.75</td>
<td>1.27</td>
</tr>
<tr>
<td>(8.8)</td>
<td>(10.0)</td>
<td>(8.0)</td>
</tr>
<tr>
<td>.293</td>
<td>.26</td>
<td>-.27</td>
</tr>
<tr>
<td>(4.2)</td>
<td>(3.7)</td>
<td>(-1.8)</td>
</tr>
<tr>
<td>-.65</td>
<td>.44</td>
<td>.45</td>
</tr>
<tr>
<td>(5.2)</td>
<td>(6.4)</td>
<td>(4.9)</td>
</tr>
<tr>
<td>-.59</td>
<td>.67</td>
<td>.60</td>
</tr>
<tr>
<td>(9.5)</td>
<td>(11.8)</td>
<td>(10.3)</td>
</tr>
</tbody>
</table>

### Significant βₖ(s)

| Significant βₖ(s) | 2,7,8,10,12 | 2,8,10 | 5,10 | 10 | 10 |

### Notes:
1. Values in parentheses are t-ratios or p-values.
2. AIC = -2(Log Likelihood) + 2M where M is the total number of parameters estimated, which helps correct for lack of parsimony.
3. It is not possible to compare s, -2(Log likelihood) or AIC for different values of the Box-Cox parameter γ.
Module 9

Segment 10

Ohio Power Example Model Comparison Using a Log Transformation and an Explanation of the Persistent ACF Spike at Lag 10
Ohio Power Consumption Model Summary Table

<table>
<thead>
<tr>
<th>Statistics 451</th>
<th>OHIO POWER EXAMPLE MODEL COMPARISON</th>
<th>Log Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box-Cox Transformation Parameter $\gamma$</td>
<td>No Transformation</td>
<td>Transformation</td>
</tr>
<tr>
<td>$1$</td>
<td>$2$</td>
<td>$3$</td>
</tr>
<tr>
<td>$d = 0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>seasonal differences</td>
<td>$D = 1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\hat{\phi}_1$</td>
<td>$0.617$</td>
<td>$0.75$</td>
</tr>
<tr>
<td>$(8.8)$</td>
<td>$(10.0)$</td>
<td>$(8.0)$</td>
</tr>
<tr>
<td>$\hat{\phi}_2$</td>
<td>$0.293$</td>
<td>$0.26$</td>
</tr>
<tr>
<td>$(4.2)$</td>
<td>$(3.7)$</td>
<td>$(-1.8)$</td>
</tr>
<tr>
<td>$\hat{\theta}_1$</td>
<td>$-0.65$</td>
<td>$0.44$</td>
</tr>
<tr>
<td>$(5.2)$</td>
<td>$(6.4)$</td>
<td>$(4.9)$</td>
</tr>
<tr>
<td>$\hat{\theta}_1$ (seasonal)</td>
<td>$-0.59$</td>
<td>$0.67$</td>
</tr>
<tr>
<td>$(9.5)$</td>
<td>$(11.8)$</td>
<td>$(9.5)$</td>
</tr>
<tr>
<td>Significant $\hat{\theta}_1$s</td>
<td>$2, 7, 8, 10, 12$</td>
<td>$2, 8, 10$</td>
</tr>
<tr>
<td>$s = \hat{\delta}_8$</td>
<td>$1.80$</td>
<td>$1.61$</td>
</tr>
<tr>
<td>AIC</td>
<td>$759.08$</td>
<td>$723.3$</td>
</tr>
<tr>
<td>$-2(\text{Log Likelihood})$</td>
<td>$755.08$</td>
<td>$717.3$</td>
</tr>
<tr>
<td>L-B $ \chi^2$ (20 dof)</td>
<td>$71.37$</td>
<td>$56.51$</td>
</tr>
<tr>
<td>$(0)$</td>
<td>$(0)$</td>
<td>$(0.01)$</td>
</tr>
<tr>
<td>Numer of parameters</td>
<td>$2$</td>
<td>$3$</td>
</tr>
</tbody>
</table>

Notes: 1) Values in parentheses are t-ratios or p-values
2) $\text{AIC} = -2(\text{Log Likelihood}) + 2M$ where $M$ is the total number of parameters estimated, which helps correct for lack of parsimony.
3) It is not possible to compare $s$, $-2(\text{log likelihood})$ or AIC for different values of the Box-Cox parameter $\gamma$. 

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Ohio Power Consumption Data

SARIMA(0,1,1)(0,1,1)_12 (Airline)

Log Transformation (Model 6) Output Part 1

esti(ohio.ts, gamma=0, model=model.pdq(d=1, q=1, D=1, Q=1))

Ohio Electric Power Consumption

Model: Component 1: ndiff= 1 ma: 1  Component 2: period= 12 ndiff= 1 ma: 1
on w= log(Billions of Kilowatt-hours)

Residuals vs. Time

Residuals vs. Fitted Values

Residual ACF
Ohio Power Consumption Data
SARIMA$(0,1,1)(0,1,1)_12$ (Airline)
No Transformation (Model 5) Output Part 1
\[
esti(\text{ohio.tsd}, \text{gamma}=1, \text{model} = \text{model.pdq(d=1, q=1, D=1, Q=1)})\]

Ohio Electric Power Consumption
Model: Component 1 :: ndiff= 1 ma: 1  Component 2 :: period= 12 ndiff= 1 ma: 1 on w= Billions of Kilowatt-hours

Residuals vs. Time

Residuals vs. Fitted Values

Residual ACF
Ohio Power Consumption Data
SARIMA\((0, 1, 1)(0, 1, 1)_{12}\) (Airline)
Log Transformation (Model 6) Output Part 2

Ohio Electric Power Consumption
Model: Component 1: ndiff= 1 ma: 1 Component 2: period= 12 ndiff= 1 ma: 1 on w= log( Billions of Kilowatt-hours )
Actual Values * * * Fitted Values * * * Future Values
* * 95 % Prediction Interval * *

Index

Billions of Kilowatt-hours

Normal Probability Plot

Residuals

Normal Scores

-3 -2 -1 0 1 2 3

-0.05 0.0 0.05 0.10
Ohio Power Consumption Data
SARIMA$(0, 1, 1)(0, 1, 1)_12$ (Airline)
No Transformation (Model 5) Output Part 2

Ohio Electric Power Consumption
Model: Component 1 :: ndiff= 1 ma: 1 Component 2 :: period= 12 ndiff= 1 ma: 1 on w= Billions of Kilowatt-hours
Actual Values * * * Fitted Values * * * Future Values
* * 95 % Prediction Interval * *
### Ohio Power Consumption Model Summary Table

<table>
<thead>
<tr>
<th>Box-Cox Transformation Parameter $\gamma$</th>
<th>OHIO POWER EXAMPLE MODEL COMPARISON</th>
<th>Log Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Transformation</td>
<td>Transformation</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\hat{\phi}_1$ (seasonal)</td>
<td>.617</td>
<td>.75</td>
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<tr>
<td></td>
<td>(8.8)</td>
<td>(10.0)</td>
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<tr>
<td>$\hat{\phi}_2$</td>
<td>.293</td>
<td>.36</td>
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<tr>
<td></td>
<td>(4.2)</td>
<td>(3.7)</td>
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<tr>
<td>$\hat{\phi}_1$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\phi}_2$ (seasonal)</td>
<td>-</td>
<td>.59</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(9.5)</td>
</tr>
<tr>
<td>Significant $p(a)$</td>
<td>2,7,8,10,12</td>
<td>2,8,10</td>
</tr>
<tr>
<td>$s = \hat{\delta}_a$</td>
<td>1.80</td>
<td>1.61</td>
</tr>
<tr>
<td>AIC</td>
<td>759.08</td>
<td>723.3</td>
</tr>
<tr>
<td>$-2$(Log Likelihood)</td>
<td>755.08</td>
<td>717.3</td>
</tr>
<tr>
<td>L-B $\chi^2$ (20df)</td>
<td>71.37</td>
<td>56.51</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>(0)</td>
</tr>
<tr>
<td>Number of parameters</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

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1) Values in parentheses are t-ratios or p-values.
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Contour Plot of $\text{SARIMA}(0, 1, 1)(0, 1, 1)_{12}$ (Airline) Model

$-2[\text{Log-likelihood}]$ Surface for the Ohio Data
Contour Plot of SARIMA(0, 1, 1)(0, 1, 1)_12 (Airline)
Model Relative Likelihood Surface for the Ohio Data
Perspective Plot of SARIMA(0, 1, 1)(0, 1, 1)_{12} (Airline) Model Relative Likelihood Surface for the Ohio Data
Ohio Power Consumption Data

SARIMA(0,1,1)(0,1,1)_{12} (Airline)

Log Transformation (Model 6) Output Part 1

esti(ohio.tsd, gamma=0, model=model.pdq(d=1, q=1, D=1, Q=1))
ACF of the Residuals for the Ohio Power using the SARIMA(0,1,1)(0,1,1)_{12} (Airline) Model

\texttt{show.acf(ohio.model6.out$resid)}