Practice Assignment 12 for Statistics 451, Spring 2008

1. Consider a monthly time series describing the sales of a company's product. For each of the following events, describe the expected effect on the time series and write down a corresponding intervention model that you would use to modify a univariate model

$$Z_t = \psi(\mathrm{B})a_t, \quad a_t \sim \mathrm{nid}(0, \sigma_a^2)$$

used to describe the process before any intervention. Indicate the nature of the input variable you use and the expected sign of any model coefficients.

   (a) The effect of a 5-month labor strike against the company.

   (b) The effect of a 5-month labor strike against a competitor.

   (c) The effect of of one-month intensive promotional effort for the product.

2. Consider the following intervention model.

$$\begin{aligned} Z_t &= \nu(\mathrm{B})I_t + n_t \\ &= \nu(\mathrm{B})I_t + \psi(\mathrm{B})a_t = \frac{\omega_0}{(1-\delta_1\mathrm{B})}I_t + \frac{1-\theta_1\mathrm{B}}{(1-\mathrm{B})}a_t, \quad a_t \sim \mathrm{nid}(0, \sigma_a^2) \end{aligned}$$

   (a) Find the unscrambled equation giving Z_t as a function of only the parameters and a finite number of lagged values of Z_t, I_t and a_t.

   (b) Briefly explain (and draw a graph to illustrate) the behavior of the transfer function

$$\nu(\mathrm{B}) = \frac{\omega_0}{(1-\delta_1\mathrm{B})}$$

   to a step input function if $\delta_1 = 0.10$ and $\omega_1 = 2$. Compare this with the response to an impulse input. [Hint: first use a geometric series expansion to determine the values of the coefficients of $\nu(\mathrm{B}) = (\nu_0 + \nu_1\mathrm{B} + \nu_2\mathrm{B}^2 + \cdots)$.] Assume that $0 < \delta_1 < 1$.

   (c) Assuming that I_t can be predicted without error, derive the prediction standard error for 1 and 2-step ahead forecasts for Z_t.

   (d) Without doing any derivations, briefly explain the effect that having to predict future values of the input I_t will have on the prediction standard error.

3. Process realizations $x_1, x_2, \ldots, x_{200}$ and $y_1, y_2, \ldots, y_{200}$ are available. Investigators feel that the transfer function model

$$y_t = \nu(\mathrm{B})x_t + n_t$$

can be used to describe the relationship between x_t and y_t. After prewhitening x_t with $\alpha_t = \psi_x(\mathrm{B})x_t$ and $\beta_t = \psi_x(\mathrm{B})y_t$, the CCF between α_t and β_t showed that $\widehat{\rho}_{\alpha\beta}(3) = .4$.

   (a) What is the interpretation of $\widehat{\rho}_{\alpha\beta}(3) = .4$?

   (b) Using the information from the CCF, what particular model is suggested for describing the relationship between x_t and y_t?

   (c) Compute the standard error of $\widehat{\rho}_{\alpha\beta}(3)$ and use it to test the null hypothesis that $\rho_{\alpha\beta}(3) = 0$. What do you conclude?

   (d) The CCF provides strong evidence that change in x_t is followed, after 3 time periods, by a corresponding change in y_t. Does this imply that changes in x_t cause changes in y_t? Explain.