

Using Accelerated Tests to Predict Service Life in Highly-Variable Environments

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Overview

- Accelerated testing—basic ideas
- Accelerated life tests and accelerated degradation tests
- Relating laboratory test results and field data and difficulties
- Deterministic and stochastic models
- Degradation and degradation models for deterministic and stochastic environments
- Relationship between degradation and failure time models
- Predictive mixtures (over time and environment) model for field failures
- Other issues and concluding remarks

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Accelerated Tests Increasingly Important

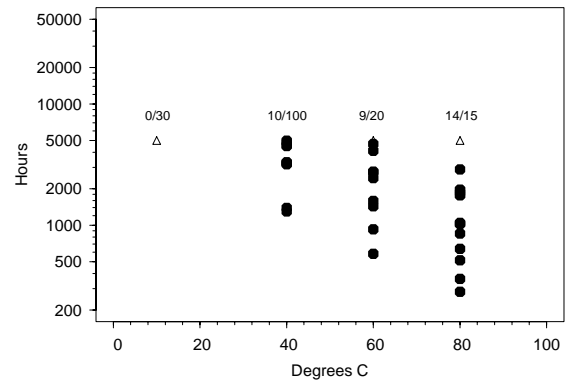
Today's manufactures need to develop newer, higher technology products in record time while improving productivity, reliability, and quality.

Important issues:

- Rapid product development.
- Rapidly changing technologies.
- More complicated products with more components.
- Higher customer expectations for better reliability.

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Device-A Hours Versus Temperature (Hooper and Amster 1990)



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The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

$$\Pr[T(\text{temp}) \leq t] = \Phi_{\text{nor}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$

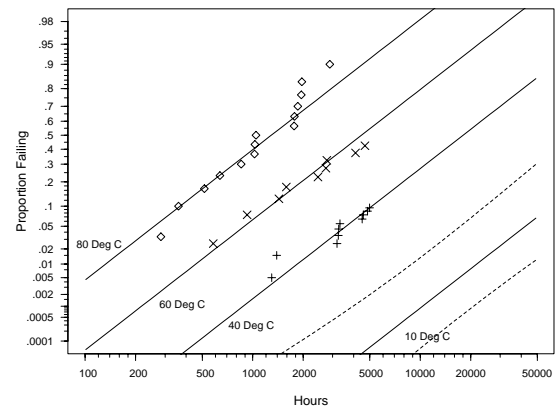
where $\mu = \beta_0 + \beta_1 x$ where

$$x = \frac{11605}{\text{temp K}} = \frac{11605}{\text{temp } ^\circ\text{C} + 273.15}$$

and $\beta_1 = E_a$ is the activation energy.

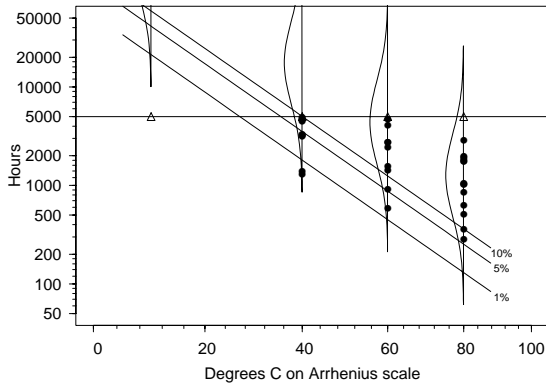
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Lognormal Multiple Probability Plot Depicting the Arrhenius-Lognormal Log-Linear Regression Model Fit to the Device-A ALT Data

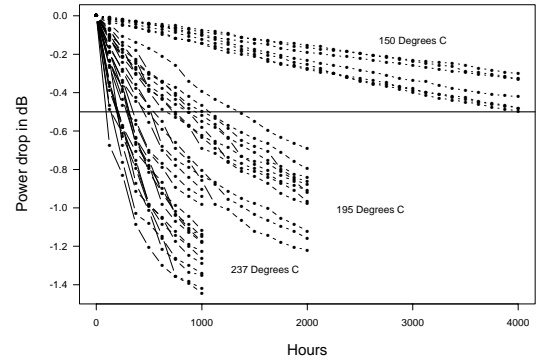


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The Arrhenius-Lognormal Log-Linear Regression Model Fit to the Device-A ALT Data



Device-B Power Drop (Meeker et al. 1998) Accelerated Degradation Test Results at 150°C, 195°C, and 237°C (Use conditions 80°C)



Difficulty Establishing Correlation Between Lab Tests and Outdoor Weathering Tests for Organic Paints and Coatings

- Inadequate control/monitoring of laboratory accelerated test conditions [e.g., $e = (\text{UV, temperature, humidity})$].
- Inadequate control/monitoring of field testing environmental conditions at outdoor exposure sites.
- Testing at excessively high levels of accelerating stresses.
- Physical/chemical models that do not provide an adequate description of the relationship between degradation rates and experimental/environmental variables.
- Prediction models and methods that do not properly account for temporal environmental variability.

Service life prediction still relies on expensive, time-consuming outdoor testing in places like Florida and Arizona.

Scale Accelerated Failure Time Model

- The particularly simple Scale Accelerated Failure Time (SAFT) model relates failure time $T(e)$ at environmental conditions e to the failure time $T(e_0)$ at environmental conditions e_0 through the relationship

$$T(e) = T(e_0)/\mathcal{AF}(e)$$

where $\mathcal{AF}(e) > 0$ is a time-invariant scale factor that depends on e and (e_0) .

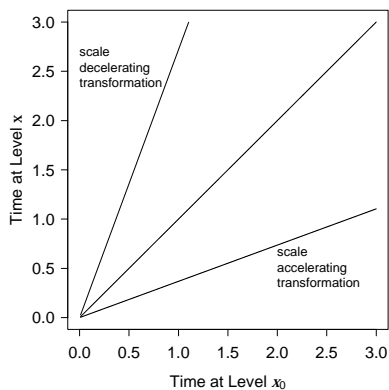
- AFT model implies proportional quantiles

$$t_p(e) = t_p(e_0)/\mathcal{AF}(e), \quad 0 < p < 1.$$

and cdfs are related by

$$\Pr [T \leq t; e] = \Pr [T \leq \mathcal{AF}(e) \times t; e_0].$$

SAFT Models Illustrating Acceleration and Deceleration.



Degradation Path

- Degradation, $\mathcal{D}(t)$, usually depends on environmental variables like UV, temp, and RH, that vary over time, say according to a multivariable profile $e(t) = [\text{UV, temp, RH, ...}]$.
- Failure usually defined as the first time at which $\mathcal{D}(t)$ crosses a threshold.
- Laboratory tests are conducted in well-controlled environments (usually holding variables like UV, temperature, and humidity constant).
- Interest often centers, however, on life in a variable environment.

Basic Approach

- Model degradation rates as a function of environmental conditions.
- Model/characterize temporal environmental variability for a given location.
- Use the environment model to drive the degradation rate model to provide a model/predictions for cumulative degradation.
- Aggregate/average product failures over multiple environments.

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Deterministic and Stochastic Models

Approach: First develop deterministic physical/chemical models. Then add random and stochastic process distributions, as needed, to account for important process variabilities (unit-to-unit, stochastic over time, or both).

There are three situations to consider:

- The environmental conditions e are constant over time.
- The environmental conditions $e = e(t)$ have a variable but deterministic path in time (i.e., a step-stress time function).
- The environmental conditions $e = e(t)$ are random in time (e.g., outdoor/real-world conditions) and the distribution of sample paths can be described by a (multivariate) stochastic process model with parameters θ_{ξ} for $e(t)$.

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Degradation in a Nonconstant Environment

- For a given environmental profile, cumulative degradation for a unit can be obtained from

$$\begin{aligned} \mathcal{D}(t) &= \int_0^t d\mathcal{D}[\tau, e(\tau)] d\tau \\ &= \int_0^t \frac{d\mathcal{D}[\tau; UV(\tau), \text{temp}(\tau), RH(\tau), \dots]}{d\tau} d\tau \end{aligned}$$

- In general, these cumulative degradation paths differ from unit to unit due to:
 - ▶ Intrinsic unit-to-unit differences (raw materials, processing differences).
 - ▶ Extrinsic differences (e.g., in environmental profiles denoted by $e(\tau)$).

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Model for Degradation of Organic Coatings and Paints

Jorgensen et al. (1996) using constant exposure accelerated laboratory tests identified a model similar to

$$\frac{d\mathcal{D}(t; L_{UV-B}, \text{temp}, RH)}{dt} = A \times L_{UV-B} \times \exp\left(-\frac{E_a}{k_B \text{temp} K}\right) \times \exp(C \times RH)$$

where $\mathcal{D}(t)$ is cumulative degradation, $d\mathcal{D}(t; L_{UV-B}, \text{temp}, RH)/dt$ is the degradation rate,

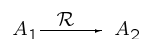
$$L_{UV-B} = L_{UV-B}(t) = \int_{290 \text{ nm}}^{320 \text{ nm}} L_{2\pi}(\lambda, t) d\lambda$$

is the instantaneous dose (in J/m^2) in the UV-B spectral range ($\lambda = 290-320 \text{ nm}$), $\text{temp} K$ is temperature Kelvin, k_B is Boltzmann's constant, and RH is relative humidity.

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Simple Example: Deterministic Degradation with Nonconstant Temperature

- For a simple first-order chemical degradation process



$$\frac{dA_1(t)}{dt} = -\mathcal{R}(\text{temp}) A_1(t) \quad \text{and} \quad \frac{dA_2(t)}{dt} = \mathcal{R}(\text{temp}) A_1(t)$$

where the reaction rate constant \mathcal{R} might have an Arrhenius relationship with temp and temp may be a function of time, t . That is $\text{temp} = \text{temp}(t)$, so \mathcal{R} can be viewed as a function of time t , say $\mathcal{R}[\text{temp}(t)]$.

- More generally, the rate constants (and thus degradation rates) could depend on other environmental variables like humidity and UV radiation characteristics (frequency and power).

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Cumulative Degradation Model

- We first discuss temperature profiles which vary through time in a deterministic manner.

- Solving the system of differential equations gives:

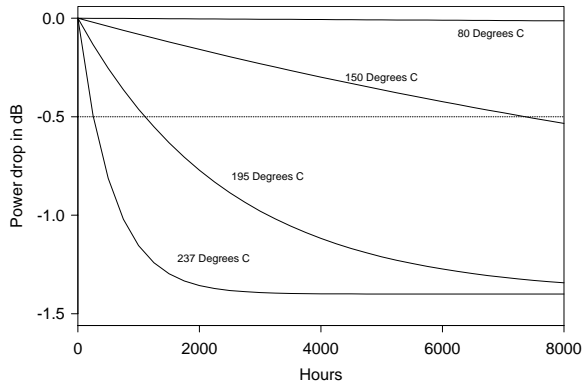
$$A_1(t) = A_1(0) \exp\left(-\int_0^t \mathcal{R}[\text{temp}(\tau)] d\tau\right)$$

$$A_2(t) = A_2(0) + A_1(0) \left[1 - \exp\left(-\int_0^t \mathcal{R}[\text{temp}(\tau)] d\tau\right)\right].$$

- Suppose that the degradation level $A_2(t)$ is observable or is proportional to an observable performance measure.
- In some cases there might be a definition of failure based on the level of $A_2(t)$. Definition may be arbitrary, but should be purposeful.

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Degradation Under Constant Temperatures



Cumulative Degradation for a One-Step Temperature Profile

- Suppose that temperature changes from $temp_1$ to $temp_2$ at time t_1 .

$$\mathcal{R}(t) = \begin{cases} \mathcal{R}_1 = \mathcal{R}(temp_1) & \text{when } 0 < t \leq t_1 \\ \mathcal{R}_2 = \mathcal{R}(temp_2) & \text{when } t_1 < t \leq t_2. \end{cases}$$

- The degradation path can be written as

$$A_2(t) = A_2(0) + A_1(0) [1 - \exp(-\mathcal{R}_1 t)], \quad 0 < t \leq t_1$$

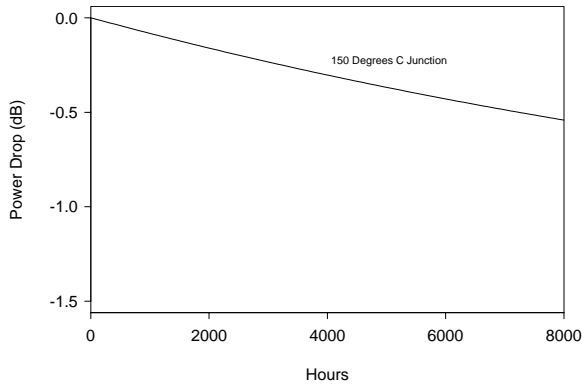
$$A_2(t) = A_2(t_1) + A_1(t_1) [1 - \exp(-\mathcal{R}_2(t - t_1))], \quad t_1 < t \leq t_2.$$

- This generalizes to other piece-wise constant profiles:

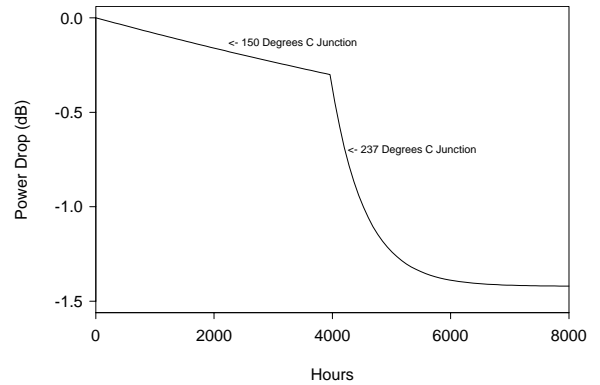
$$A_2(t) = A_2(t_{i-1}) + A_1(t_{i-1}) [1 - \exp(-\mathcal{R}_i(t - t_{i-1}))], \quad t_{i-1} < t \leq t_i,$$

where $i = 1, 2, \dots$, $\mathcal{R}_i = \mathcal{R}(temp_i)$, $temp_i$ is the temperature between t_{i-1} and t_i , and $t_0 = 0$.

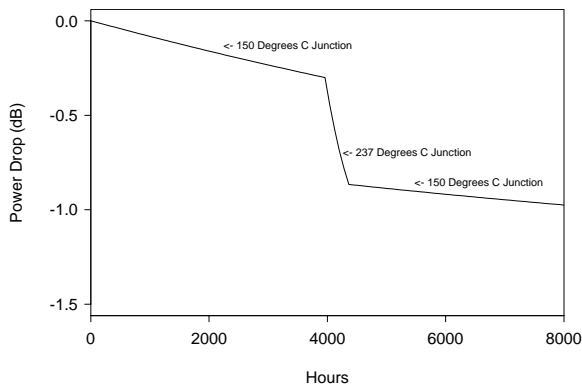
Degradation with Constant Temperature



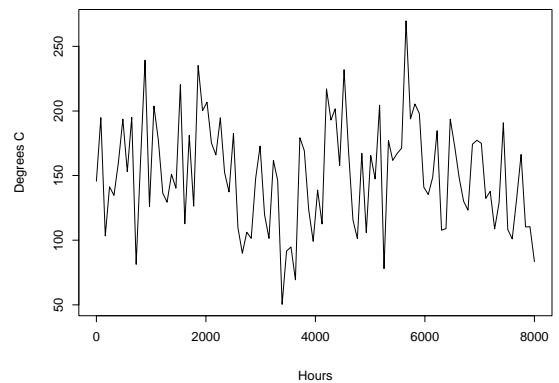
Degradation with a Change in Temperature



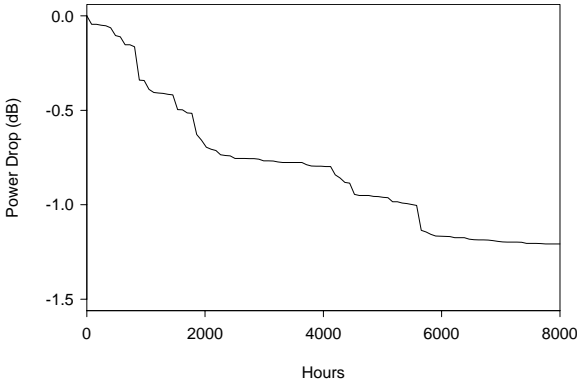
Degradation with a Several Changes in Temperature



Random Temperature Profile (Independent Normal with $\mu = 150$, $\sigma = 40$)

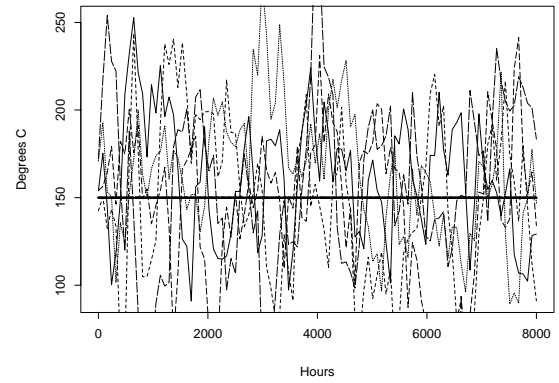


Degradation with a Random Temperature Profile



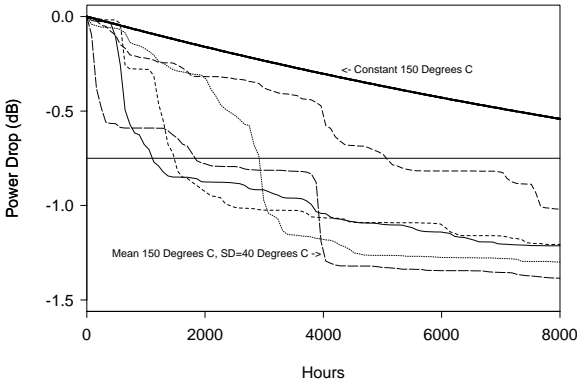
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Temperature profile $\rho_1 = 0.7$ AR(1) realizations with $\mu = 150^\circ\text{C}$ and $\sigma = 40^\circ\text{C}$.



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$\mathcal{D}(t)$ Power-Drop Cumulative Degradation Paths.



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Population Failure Probability with Stochastic Environmental Profiles

- For the entire population of units, the failure probability is

$$\begin{aligned} \Pr[A_2(t) \leq A_{2f}; \theta_\beta, \theta_\xi] &= F(t; \theta_\beta, \theta_\xi) \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Pr[A_2(t) \leq A_{2f} | \beta; \theta_\xi] f(\beta; \theta_\beta) d\beta_1 d\beta_2 \end{aligned}$$

- In general, the failure-time distribution [failure time defined as the smallest $t \geq 0$ for which $\mathcal{D}(t) \leq \mathcal{D}_f$] is

$$\begin{aligned} F(t; \theta_\beta, \theta_\xi) &= \Pr(T \leq t) \\ &= \int_{\beta} \Pr[\mathcal{D}(t) \leq \mathcal{D}_f; \beta, \theta_\xi] f(\beta; \theta_\beta) d\beta \end{aligned}$$

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Unit-to-Unit or Region-to-Region Mixtures of Environmental Conditions

To simplify presentation, we consider a model for a population of units that was placed into service over a relatively short period of time (perhaps one month).

- The population of units in the field can be subdivided into k subpopulations of units, according to the environmental conditions to which they are exposed.
- There are $n_i = n(\theta_{\xi_i})$ units in subpopulation i having environmental conditions described by θ_{ξ_i} , $i = 1, 2, \dots, k$.
- The total number of units in the field is $n = \sum_{i=1}^k n_i$.
- The relative frequency (or proportion) of units at conditions θ_{ξ_i} will be denoted by $f_i = n_i/n$, $i = 1, \dots, k$.

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Mixture Population Failure-Time Distribution

- For subpopulation i with environment conditions θ_{ξ_i} occurring with relative frequency f_i , $i = 1, \dots, k$

$$\Pr(T \leq t) = \sum_{i=1}^k \Pr[T \leq t; \theta_{\xi_i}] \times f_i$$

where $n_i = n \times f_i$ is the number of units on test at environmental conditions θ_{ξ_i} .

- Suppose that n is the total number of exposed units. For the general time transformation model, the expected number of units failed by time t is

$$E[N(t)] = n \times \Pr(T \leq t)$$

- Aggregate over multiple time cohorts for product entering the field over time.

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Other Issues

- More complicated cumulative degradation models
 - ▶ Multi-step reactions may (probably will have) different activation energies.
 - ▶ May encounter path dependence in which degradation rate depends on history (can detect with step stress experiments)
- Effects of other environmental factors (e.g., acid rain, presence of moisture, mechanical stresses, etc.)
- Methods of time series modeling and simulation.
- Feasible computational methods.

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- Prediction intervals to describe uncertainty in generated predictions.

Concluding Remarks

- Accelerated testing provides interesting scientific and statistical challenges.
- Modeling (physical and statistical) is a difficult but essential part of accelerated testing.
- Using degradation modeling and data will be important for most applications.
- In highly variable environments, average environmental conditions will not be sufficient to predict product life.

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