Using Accelerated Tests to Predict Service Life in Highly-Variable Environments

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November 14, 1999

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Overview

- Accelerated testing—basic ideas
- Accelerated life tests and accelerated degradation tests
- Relating laboratory test results and field data and difficulties
- Deterministic and stochastic models
- Degradation and degradation models for deterministic and stochastic environments
- Relationship between degradation and failure time models
- Predictive mixtures (over time and environment) model for field failures
- Other issues and concluding remarks

Accelerated Tests Increasingly Important

Today’s manufactures need to develop newer, higher technology products in record time while improving productivity, reliability, and quality.

Important issues:
- Rapid product development.
- Rapidly changing technologies.
- More complicated products with more components.
- Higher customer expectations for better reliability.

The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

\[ \Pr[T(\text{temp}) \leq t] = \Phi_{\text{nor}} \left( \frac{\log(t) - \mu}{\sigma} \right) \]

where \( \mu = \beta_0 + \beta_1x \) where

\[ x = \frac{11605}{\text{temp}K} = \frac{11605}{\text{temp}^\circ C + 273.15} \]

and \( \beta_1 = E_a \) is the activation energy.
The Arrhenius-Lognormal Log-Linear Regression Model Fit to the Device-A ALT Data

![Graph showing Arrhenius-Lognormal Log-Linear Regression Model](image)

Difficulty Establishing Correlation Between Lab Tests and Outdoor Weathering Tests for Organic Paints and Coatings

- Inadequate control/monitoring of laboratory accelerated test conditions [e.g., $e = (UV, \text{temperature}, \text{humidity})$].
- Inadequate control/monitoring of field testing environmental conditions at outdoor exposure sites.
- Testing at excessively high levels of accelerating stresses.
- Physical/chemical models that do not provide an adequate description of the relationship between degradation rates and experimental/environmental variables.
- Prediction models and methods that do not properly account for temporal environmental variability.

Service life prediction still relies on expensive, time-consuming outdoor testing in places like Florida and Arizona.

Device-B Power Drop (Meeker et al. 1998) Accelerated Degradation Test Results at 150°C, 195°C, and 237°C (Use conditions 80°C)

![Graph showing Device-B Power Drop](image)

Scale Accelerated Failure Time Model

- The particularly simple Scale Accelerated Failure Time (SAFT) model relates failure time $T(e)$ at environmental conditions $e$ to the failure time $T(e_0)$ at environmental conditions $e_0$ through the relationship
  
  $$T(e) = T(e_0) \cdot A^F(e)$$

  where $A^F(e) > 0$ is a time-invariant scale factor that depends on $e$ and $(e_0)$.

- AFT model implies proportional quantiles
  
  $$t_p(e) = t_p(e_0) \cdot A^F(e), \quad 0 < p < 1.$$

  and cdfs are related by
  
  $$Pr[T \leq t; e] = Pr[T \leq A^F(e) \cdot t; e_0].$$

Degradation Path

- Degradation, $D(t)$, usually depends on environmental variables like $UV$, $\text{temp}$, and $\text{RH}$, that vary over time, say according to a multivariable profile $e(t) = [UV, \text{temp}, \text{RH}, \ldots]$.

- Failure usually defined as the first time at which $D(t)$ crosses a threshold.

- Laboratory tests are conducted in well-controlled environments (usually holding variables like $UV$, temperature, and humidity constant).

- Interest often centers, however, on life in a variable environment.
Basic Approach

- Model degradation rates as a function of environmental conditions.
- Model/characterize temporal environmental variability for a given location.
- Use the environment model to drive the degradation rate model to provide a model/predictions for cumulative degradation.
- Aggregate/average product failures over multiple environments.

Deterministic and Stochastic Models

Approach: First develop deterministic physical/chemical models. Then add random and stochastic process distributions, as needed, to account for important process variabilities (unit-to-unit, stochastic over time, or both).

- The environmental conditions $e$ are constant over time.
- The environmental conditions $e = e(t)$ have a variable but deterministic path in time (i.e., a step-stress time function).
- The environmental conditions $e = e(t)$ are random in time (e.g., outdoor/real-world conditions) and the distribution of sample paths can be described by a (multivariate) stochastic process model with parameters $\theta_e$ for $e(t)$.

Degradation in a Nonconstant Environment

- For a given environmental profile, cumulative degradation for a unit can be obtained from
  
  $$ D(t) = \int_0^t dD[\tau, e(\tau)] d\tau $$
  
  $$ = \int_0^t dD[\tau; \text{temp}(\tau), \text{RH}(\tau), \ldots] d\tau $$

- In general, these cumulative degradation paths differ from unit to unit due to:
  
  - Intrinsic unit-to-unit differences (raw materials, processing differences).
  - Extrinsic differences (e.g., in environmental profiles denoted by $e(\tau)$).

Model for Degradation of Organic Coatings and Paints

Jorgensen et al. (1996) using constant exposure accelerated laboratory tests identified a model similar to

$$ \frac{dD(t; \text{UV-B}, \text{temp}, \text{RH})}{dt} = A \times \text{UV-B} \times \exp\left( -\frac{E_a}{k_B \text{temp} R} \right) \times \exp(C \text{RH}) $$

where $D(t)$ is cumulative degradation, $dD(t; \text{UV-B}, \text{temp}, \text{RH})/dt$ is the degradation rate,

$$ L_{\text{UV-B}} = L_{\text{UV-B}}(t) = \int_{290 \text{ nm}}^{320 \text{ nm}} L_2(\lambda, t) d\lambda $$

is the instantaneous dose (in J/m²) in the UV-B spectral range ($\lambda = 290-320$ nm), temp K is temperature Kelvin, $k_B$ is Boltzmann’s constant, and RH is relative humidity.

Simple Example: Deterministic Degradation with Nonconstant Temperature

- For a simple first-order chemical degradation process

  $$ A_1 \overset{R}{\rightarrow} A_2 $$

  $$ \frac{dA_1(t)}{dt} = -R(\text{temp}) A_1(t) \quad \text{and} \quad \frac{dA_2(t)}{dt} = R(\text{temp}) A_1(t) $$

  where the reaction rate constant $R$ might have an Arrhenius relationship with temp and temp may be a function of time, $t$. That is temp = temp$(t)$, so $R$ can be viewed as a function of time, say $R[\text{temp}(t)]$.

- More generally, the rate constants (and thus degradation rates) could depend on other environmental variables like humidity and UV radiation characteristics (frequency and power).

Cumulative Degradation Model

- We first discuss temperature profiles which vary through time in a deterministic manner.

- Solving the system of differential equations gives:

  $$ A_1(t) = A_1(0) \exp\left( -\int_0^t R[\text{temp}(\tau)] d\tau \right) $$

  $$ A_2(t) = A_2(0) + A_1(0) \left[ 1 - \exp\left( -\int_0^t R[\text{temp}(\tau)] d\tau \right) \right] $$

- Suppose that the degradation level $A_2(t)$ is observable or is proportional to an observable performance measure.

- In some cases there might be a definition of failure based on the level of $A_2(t)$. Definition may be arbitrary, but should be purposeful.
Suppose that temperature changes from $\text{temp}_1$ to $\text{temp}_2$ at time $t_1$.

$$ R(t) = \begin{cases} R_1 = R(\text{temp}_1) & \text{when } 0 < t \leq t_1 \\ R_2 = R(\text{temp}_2) & \text{when } t_1 < t \leq t_2. \end{cases} $$

The degradation path can be written as

$$ A_2(t) = A_2(0) + A_1(0)[1 - \exp(-R_1 t)], \quad 0 < t \leq t_1 $$

$$ A_2(t) = A_2(t_1) + A_1(t_1)[1 - \exp(-R_2 (t - t_1))], \quad t_1 < t \leq t_2. $$

This generalizes to other piece-wise constant profiles:

$$ A_2(t) = A_2(t_{i-1}) + A_1(t_{i-1})[1 - \exp(-R_i (t - t_{i-1}))], \quad t_{i-1} < t \leq t_i, $$

where $i = 1, 2, \ldots$, $R_i = R(\text{temp}_i)$, $\text{temp}_i$ is the temperature between $t_{i-1}$ and $t_i$, and $t_0 = 0$. 

\[\]
Degradation with a Random Temperature Profile

Temperature profile $\mu = 0.7$ AR(1) realizations with $\mu = 150^\circ C$ and $\sigma = 40^\circ C$.

$D(t)$ Power-Drop Cumulative Degradation Paths.

Population Failure Probability with Stochastic Environmental Profiles

- For the entire population of units, the failure probability is
  \[ \Pr [A_2(t) \leq A_2; \theta_\beta \theta_\xi] = F(t; \theta_\beta \theta_\xi) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Pr [A_2(t) \leq A_2; \beta, \theta_\xi] f(\beta; \theta_\beta) d\beta d\theta_\xi. \]

- In general, the failure-time distribution (failure time defined as the smallest $t \geq 0$ for which $D(t) \leq D_t$) is
  \[ F(t; \theta_\beta \theta_\xi) = \Pr(T \leq t) = \int_{\beta} \Pr[D(t) \leq D_t; \beta, \theta_\xi] f(\beta; \theta_\beta) d\beta. \]

Unit-to-Unit or Region-to-Region Mixtures of Environmental Conditions

To simplify presentation, we consider a model for a population of units that was placed into service over a relatively short period of time (perhaps one month).

- The population of units in the field can be subdivided into $k$ subpopulations of units, according to the environmental conditions to which they are exposed.
- There are $n_i = n(\theta_\xi_i)$ units in subpopulation $i$ having environmental conditions described by $\theta_\xi_i$, $i = 1, 2, \ldots, k$.
- The total number of units in the field is $n = \sum_{i=1}^{k} n_i$.
- The relative frequency (or proportion) of units at conditions $\theta_\xi_i$ will be denoted by $f_i = n_i/n$, $i = 1, \ldots, k$.

Mixture Population Failure-Time Distribution

- For subpopulation $i$ with environment conditions $\theta_\xi_i$ occurring with relative frequency $f_i, i = 1, \ldots, k$,
  \[ \Pr(T \leq t) = \sum_{i=1}^{k} \Pr(T \leq t; \theta_\xi_i) \times f_i \]
  where $n_i = n \times f_i$ is the number of units on test at environmental conditions $\theta_\xi_i$.

- Suppose that $n$ is the total number of exposed units. For the general time transformation model, the expected number of units failed by time $t$ is
  \[ E[N(t)] = n \times \Pr(T \leq t) \]
  where $N(t)$ is the number of failures by time $t$.

- Aggregate over multiple time cohorts for product entering the field over time.
Other Issues

- More complicated cumulative degradation models
  - Multi-step reactions may (probably will have) different activation energies.
  - May encounter path dependence in which degradation rate depends on history (can detect with step stress experiments)
- Effects of other environmental factors (e.g., acid rain, presence of moisture, mechanical stresses, etc.)
- Methods of time series modeling and simulation.
- Feasible computational methods.

Concluding Remarks

- Accelerated testing provides interesting scientific and statistical challenges.
- Modeling (physical and statistical) is a difficult but essential part of accelerated testing.
- Using degradation modeling and data will be important for most applications.
- In highly variable environments, average environmental conditions will not be sufficient to predict product life.