

Use of Sensitivity Analysis to Assess the Effect of Model Uncertainty in Analyzing Accelerated Life Test Data

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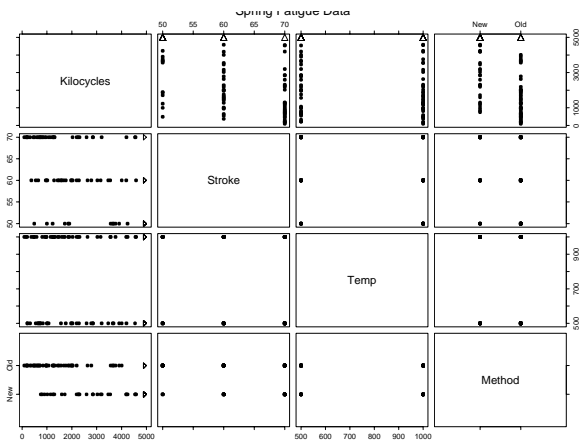
August 3, 2001

Work done in collaboration with Luis Escobar, Louisiana State University

Overview

- Accelerated life tests
- Spring accelerated life test data
- Graphical analysis
- Acceleration model and analysis
- Inference at use conditions and assessment of sampling error
- Using sensitivity analysis to assess possible model error
- Software
- Concluding remarks

Spring Accelerated Life Test Data Pairs Plot



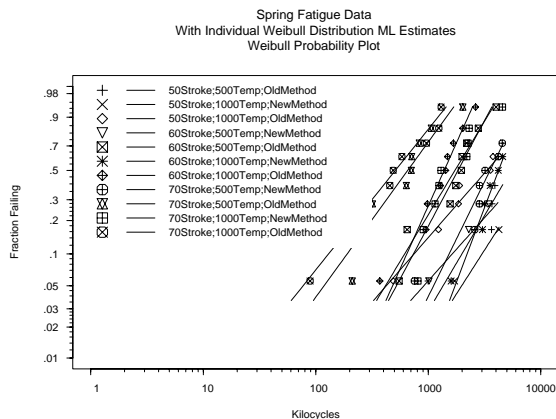
Weibull Distribution Log Location Scale Family Member

$$F(t) = \Pr(T \leq t) = 1 - \exp \left[- \left(\frac{t}{\eta} \right)^\beta \right]$$

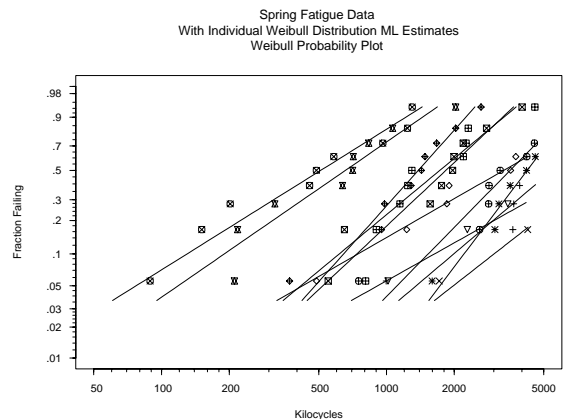
$$= \Phi_{\text{sev}} \left[\frac{\log(t) - \mu}{\sigma} \right], \quad t > 0$$

where $\mu = \log(\eta)$, $\sigma = 1/\beta$, and $\Phi_{\text{sev}}(z) = 1 - \exp[-\exp(z)]$.

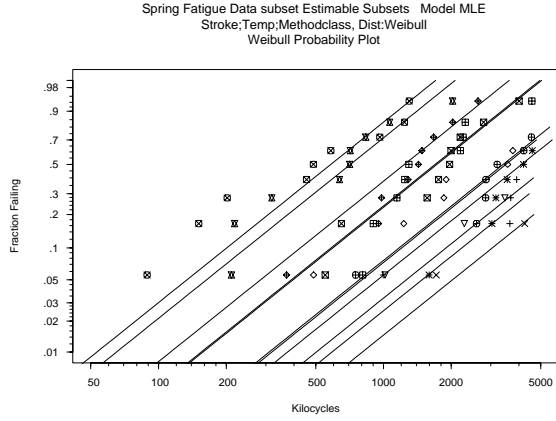
Spring Accelerated Life Test Data Weibull Multiple Probability Plot Individual ML Estimates of $F(t)$



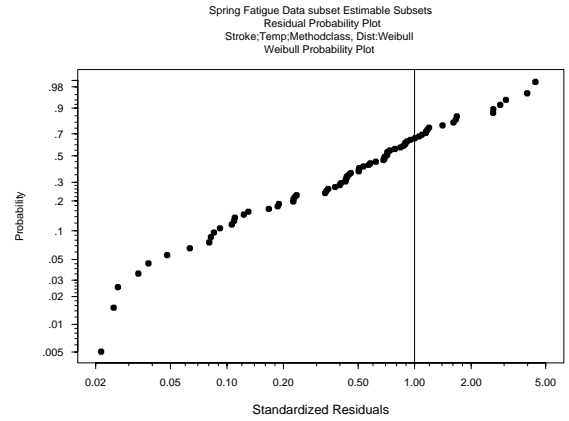
Spring Accelerated Life Test Data Weibull Multiple Probability Plot Individual ML Estimates of $F(t)$



**Weibull Multiple Probability Plot
Individual ML Estimates of $F(t)$,
Common Weibull Shape Parameter (Floating Scale)**



**Spring Accelerated Life Test Data
Weibull Residual Probability Plot
Floating Scale Model**



**Spring Accelerated Life Test
Weibull Distribution Response Surface Model**

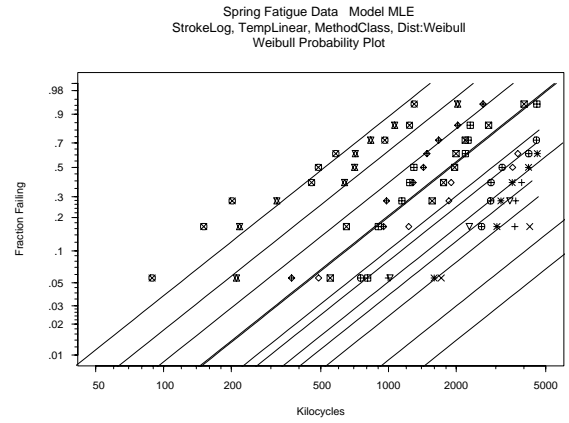
$$F(t; \mu, \sigma) = \Phi_{\text{sev}} \left[\frac{\log(t) - \mu}{\sigma} \right], \quad t > 0$$

$$\mu = \beta_0 + \beta_1 \log(\text{Stroke}) + \beta_2 \text{Temp} + \beta_3 \text{Method}$$

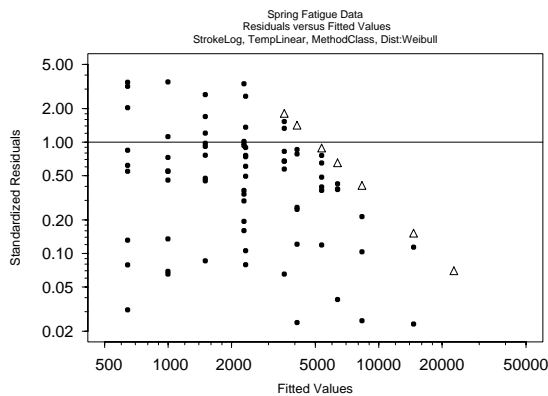
$$\sigma = \text{constant}$$

where Method = 0 for New and Method = 1 for Old.

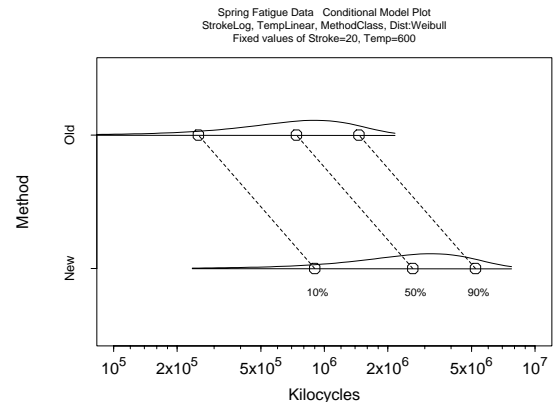
**Spring Accelerated Life Test Data
Weibull Multiple Probability Plot
ML Response Surface Estimates of $F(t)$**



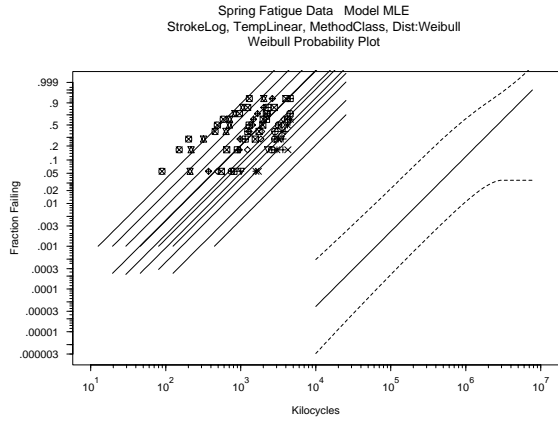
**Spring Accelerated Life Test Data
Weibull Distribution Response Surface Model
Residuals Versus Fitted Values**



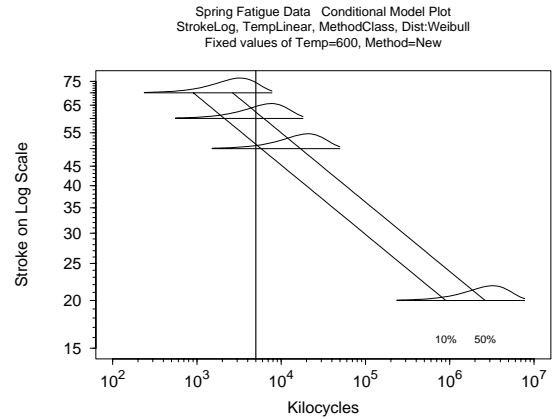
**Spring Accelerated Life Test Data
Conditional Model Plot
Spring Life versus Method**



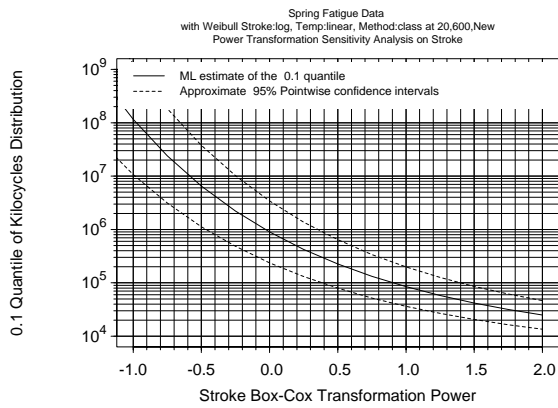
**Weibull Multiple Probability Plot
Response Surface ML Estimates
Extrapolation in Stroke Displacement**



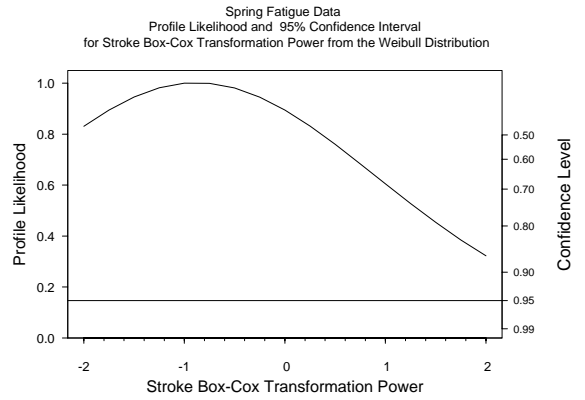
**Spring Accelerated Life Test Data
Conditional Model Plot
Spring Life versus Stroke Displacement**



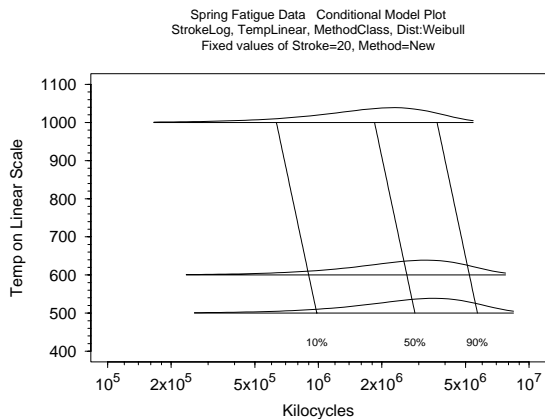
**0.10 Quantile of Spring Life versus
Stroke Displacement Box-Cox Parameter
with 95% Confidence Limits**



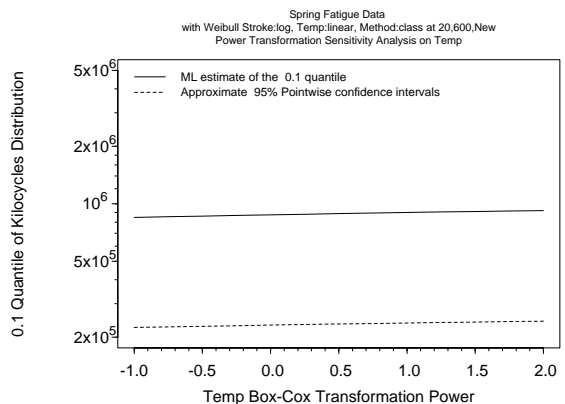
**Profile Likelihood
Stroke Box-Cox Transformation Parameter
Spring Fatigue Life Model**



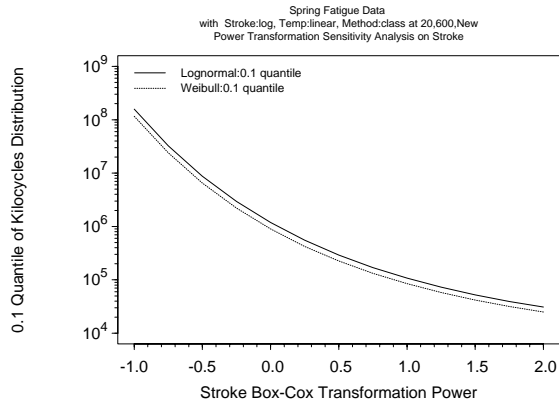
**Conditional Model Plot
Spring Life versus Processing Temperature**



**0.10 Quantile of Spring Life versus
Temperature Box-Cox Parameter
with 95% Confidence Limits**



0.10 Quantile of Spring Life versus Stroke Displacement Box-Cox Parameter Comparing Weibull and Lognormal Distributions



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SPLIDA (S-PLUS Life Data Analysis)

- Collections of S-PLUS functions for reliability data analysis
- Runs on S-PLUS Windows versions 4.5 and 2000
- Provides ability to do about 95% of the examples in Meeker and Escobar (1998) plus new capabilities
- Graphical user interface
- Available for download from www.public.iastate.edu/~wqmeeker

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Concluding Remarks

- Accelerated life testing is a critically important tool for the design of high-reliability products
- Extrapolation is required in accelerated testing
- Fundamental knowledge of the mechanisms underlying failure modes is important
- When fundamental knowledge of the mechanisms underlying is not available, sensitivity analysis and conservative design decisions are required
- Software is needed to make sensitivity analyses easy to perform

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