Use of Sensitivity Analysis to Assess the Effect of Model Uncertainty in Analyzing Accelerated Life Test Data

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Overview

• Accelerated life tests
• Spring accelerated life test data
• Graphical analysis
• Acceleration model and analysis
• Inference at use conditions and assessment of sampling error
• Using sensitivity analysis to assess possible model error
• Software
• Concluding remarks

Spring Accelerated Life Test Data

Pairs Plot

Weibull Distribution

Log Location Scale Family Member

\[ F(t) = \Pr(T \leq t) = 1 - \exp \left( -\left( \frac{t}{\eta} \right)^{\beta} \right) \]

\[ = \Phi_{\text{sev}} \left( \frac{\log(t) - \mu}{\sigma} \right), \quad t > 0 \]

where \( \mu = \log(\eta) \), \( \sigma = 1/\beta \), and \( \Phi_{\text{sev}}(z) = 1 - \exp[-\exp(z)] \).
Spring Accelerated Life Test Data
Weibull Distribution Response Surface Model

Residuals Versus Fitted Values

Spring Fatigue Data: Residuals versus Fitted Values
Stroke, Log, Temp, Linear, Method, Class, Dist: Weibull
Weibull Probability Plot

Standardized Residuals

Method

New
Old

10%
50%
90%

Spring Fatigue Data: Conditional Model Plot
Spring Life versus Method

Spring Fatigue Data: Conditional Model Plot
Stroke, Log, Temp, Linear, Method, Class, Dist: Weibull
Fixed values of Stroke=20, Temp=600

Standardized Residuals

Method

New
Old

10%
50%
90%

Kilocycles

10^5
2 \times 10^5
5 \times 10^5
2 \times 10^6
5 \times 10^6
10^7

Fitted Values

500 1000 2000 5000 10000 20000 50000

Standardized Residuals

0.02 0.05 0.10 0.20 0.50 1.00 2.00 5.00

Kilocycles

0.01 0.02 0.03 0.05 0.1 0.2 0.3 0.5 0.7 0.9 0.98

Fraction Failing

Kilocycles

.01 .02 .03 .05 .1 .2 .3 .5 .7 .9

50 100 200 500 1000 2000 5000

Spring Fatigue Data subset Estimable Subsets
Model MLE
Stroke, Temp, Method, Class, Dist: Weibull
Weibull Probability Plot

Weibull Multiple Probability Plot
Individual ML Estimates of $F(t)$
Common Weibull Shape Parameter (Floating Scale)

$F(t; \mu, \sigma) = \Phi_{\text{sev}} \left[ \frac{\log(t) - \mu}{\sigma} \right], \quad t > 0$

$\mu = \beta_0 + \beta_1 \log(\text{Stroke}) + \beta_2 \text{Temp} + \beta_3 \text{Method}$

$\sigma = \text{constant}$

where Method = 0 for New and Method = 1 for Old.
Spring Fatigue Data  Model MLE  StrokeLog, Temp:Linear, Method:Class, Dist:Weibull  Weibull Probability Plot

Spring Fatigue Data  Conditional Model Plot  Spring Life versus Stroke Displacement

Weibull Multiple Probability Plot  Response Surface ML Estimates  Extrapolation in Stroke Displacement

Spring Fatigue Data  Conditional Model Plot  StrokeLog, Temp:Linear, Method:Class, Dist:Weibull  Fixed values of Temp=600, Method=New

0.10 Quantile of Spring Life versus Stroke Displacement Box-Cox Parameter with 95% Confidence Limits

0.10 Quantile of Spring Life versus Temperature Box-Cox Parameter with 95% Confidence Limits

Profile Likelihood  Stroke Box-Cox Transformation Parameter  Spring Fatigue Life Model

Profile Likelihood and 95% Confidence Interval for Stroke Box-Cox Transformation Power from the Weibull Distribution

Confidence Level

Conditional Model Plot  Spring Life versus Processing Temperature

Conditional Model Plot  Spring Life versus Stroke Displacement

Power Transformation Sensitivity Analysis on Stroke

Spring Fatigue Data  Conditional Model Plot  StrokeLog, Temp:Linear, Method:Class at 20,600, New  Power Transformation Sensitivity Analysis on Stroke
Concluding Remarks

- Accelerated life testing is a critically important tool for the design of high-reliability products
- Extrapolation is required in accelerated testing
- Fundamental knowledge of the mechanisms underlying failure modes is important
- When fundamental knowledge of the mechanisms underlying is not available, sensitivity analysis and conservative design decisions are required
- Software is needed to make sensitivity analyses easy to perform