

## A Methodology for Predicting Probability of Detection for Ultrasonic Testing

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## Motivation

- FAA Engine Titanium Consortium goal to improve and quantify POD inspection for detecting hard alpha
- Empirical determination of POD for new situations is not always practical due to lack of time, test specimens (especially data on real flaws), and other limited resources.
- Physical models of ultrasonic signals provide a useful alternative to empirical determination but may not account for all factors and sources of variability, particularly for real flaws (e.g., hard alpha inclusions).
- Combining physical models of ultrasonic signals with an empirical adjustment has the potential to provide a workable methodology

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## Overview

- Factors and sources of variability in UT inspection.
- Comparison of physical model-based distribution and empirical distribution.
- Probability integral transform.
- Adjustment to account for the differences between the model and the empirical distributions.
- Example based on synthetic hard alpha flaws in titanium.

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## Probability Model for a UT Flaw Signal

- Probability distribution for a **flaw signal**  $Y$  is

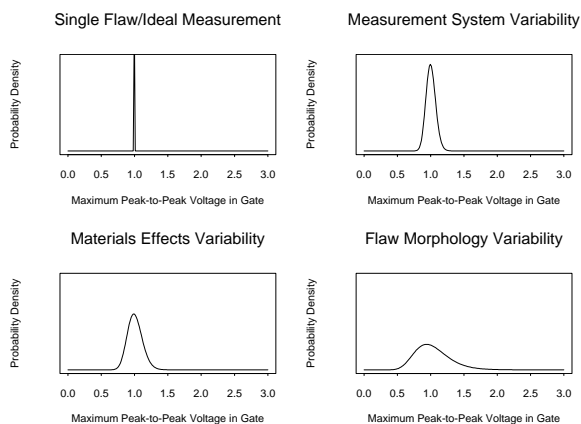
$$\Pr(Y \leq y; \underline{x}) = F_Y(y; \underline{x})$$

where  $\underline{x} = (\underline{x}_{\text{NDE}}, \underline{x}_{\text{PART}}, \underline{x}_{\text{FLAW}})$ .

- ▶  $\underline{x}_{\text{NDE}}$  contains NDE inspection system factors like probe characteristics, focus depth, scan increment, pulse rate, etc.
- ▶  $\underline{x}_{\text{PART}}$  contains PART factors like part geometry, type of material being inspected, surface roughness, etc.
- ▶  $\underline{x}_{\text{FLAW}}$  contains flaw factors like size, depth, density, shape, composition, and degree of voiding/cracking.

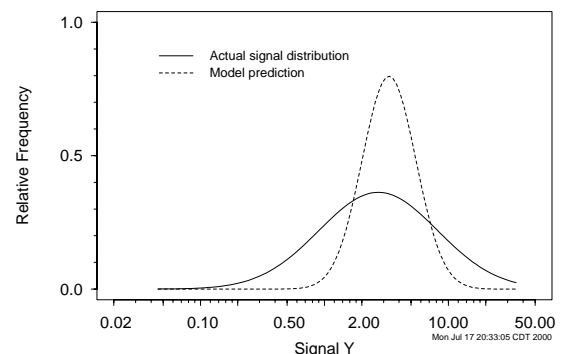
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## Sources of Variability in Nondestructive Evaluation.



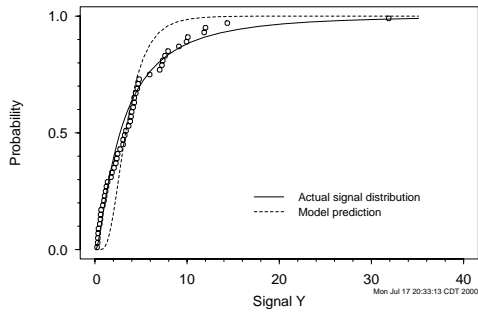
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## Systematic Differences Between the Model-Predicted Signal pdf $f_{Y^*}(y; \underline{x})$ and the actual pdf $f_Y(y; \underline{x})$ for a given $\underline{x}$ .



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**Comparison Between Actual  $F_Y(y; \underline{x})$  and Model-Predicted cdf  $F_{Y^*}(y; \underline{x})$ . Points Represent Empirical Estimate of  $F_Y(y; \underline{x})$  from Simulated Data**



**Probability Integral Transform**

- Suppose that the random variable  $Y$  has a cumulative distribution function  $F_Y(y; \underline{x})$ .

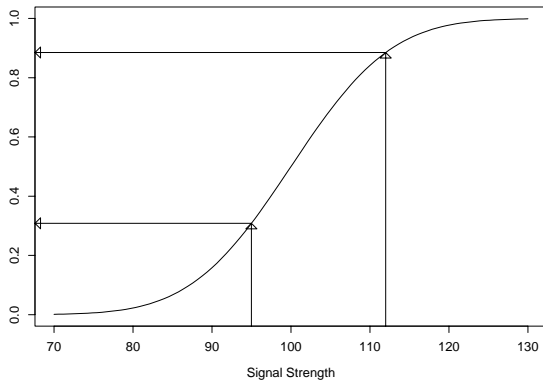
- Then

$$U = F_Y(Y; \underline{x})$$

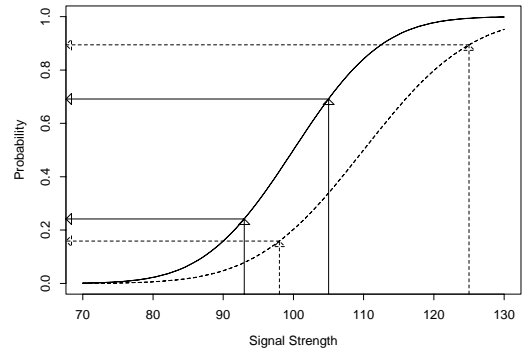
has a Uniform distribution between 0 and 1.

This is a well-known result in probability and mathematical statistics.

**Illustration of the Probability Integral Transform with a Single  $\underline{x}$**



**Illustration of the Probability Integral Transform with Two  $\underline{x}$ 's**

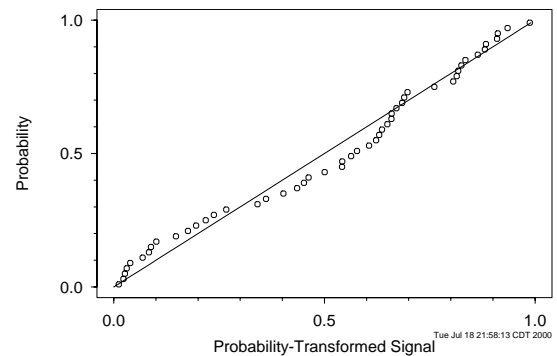


**Empirical Probability Integral Transform**

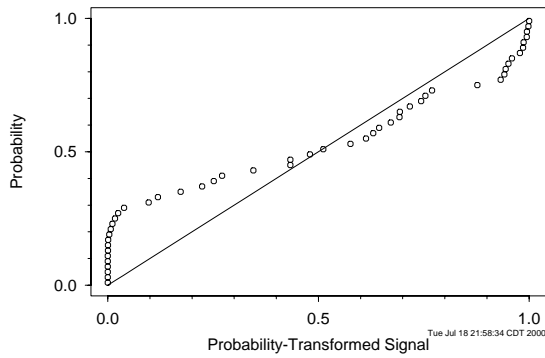
Suppose we have signal values  $y_1, y_2, \dots, y_n$  (along with the corresponding, possibly different,  $\underline{x}_1, \underline{x}_2, \dots, \underline{x}_n$  values). Then compute  $u_1, u_2, \dots, u_n$  from UT physical model as  $u_i = F_{Y^*}(y_i; \underline{x}_i)$ .

- If the UT physical model fits well, then the  $u_1, u_2, \dots, u_n$  values will lie close to the diagonal.
- If the  $u_1, u_2, \dots, u_n$  values do not lie close to the diagonal, the systematic deviation suggests how to make an adjustment to the UT physical model.
- The empirical probability integral transform provides
  - ▶ A means of assessing model goodness of fit.
  - ▶ An method of model adjustment when the model does not fit.

**$F_Y(y)$  and Data Probability-Integral Transformed with the Actual  $F_Y(y)$  Distribution**



$F_Y(y)$  and Data Probability-Integral Transformed with an Inadequate UT Physical Model



Extension of the Probability Integral Transform

- Suppose that the random variable  $Y$  has a cumulative distribution function  $F_Y(y; \underline{x})$ .

- Then

$$U = F_Y(Y; \underline{x})$$

has a Uniform distribution between 0 and 1.

- The transformation

$$Z = \Phi^{-1}(U) = \Phi^{-1}[F_Y(Y; \underline{x})]$$

has a standard normal (Gaussian) distribution with mean  $\mu = 0$  and standard deviation  $\sigma = 1$ .

Steps in the Analysis

- Compute

$$z_i = \Phi^{-1}(u_i) = \Phi^{-1}[F_{Y^*}(y_i; \underline{x}_i)], i = 1, \dots, n$$

- If the UT physical model  $F_{Y^*}(y; \underline{x})$  fits the data well, then the  $z_i, i = 1, \dots, n$  should look like a standard normal (Gaussian) distribution with parameters  $\mu = 0$  and  $\sigma = 1$ .

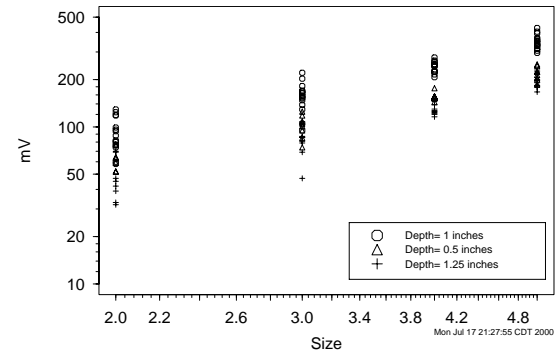
- Otherwise obtain an adjusted distribution by estimating  $\mu$  and  $\sigma$  from  $z_i, i = 1, \dots, n$  and using

$$\Pr(Y \leq y; \underline{x}) = F_{y^{**}}(y; \underline{x}) = \Phi \left[ \frac{\Phi^{-1}(F_{Y^*}(y; \underline{x})) - \hat{\mu}}{\hat{\sigma}} \right]$$

as the predicted signal-distribution (input to POD computations).

Measured UT Response from Synthetic Hard Alpha Flaws from Chiou et al. (1996, 1997) .

UT responses from SHA flaws



Physical Signal-Prediction Model

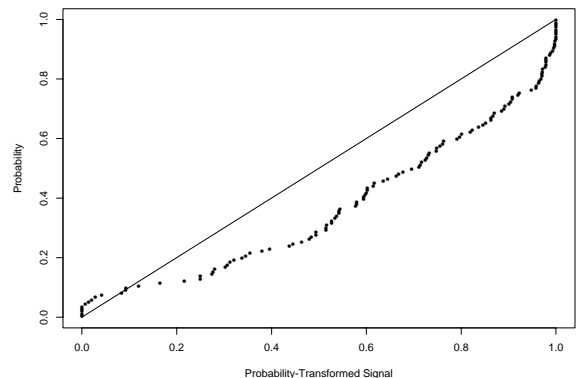
The physical model approximation cumulative distribution

$$F_{Y^*}(y; \underline{x}) = \dots$$

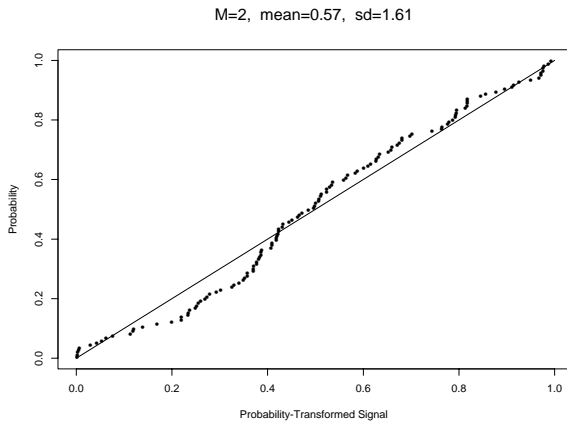
where  $\underline{x} = (a, d)$  reflects flaw size and depth (e.g., from the model of Yalda, Margetan, and Thompson (1998)) based on Rician distributions to predict the distributions of ultrasonic flaw signals in the presence of backscattered noise).

Physical Model Probability-Integral-Transformed Synthetic Hard Alpha Data.

M=2, mean=0.57, sd=1.61



### Adjusted-Model Probability-Integral-Transformed Synthetic Hard Alpha Data.



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### Computation of $POD(a)$

- The adjusted probability distribution model for **flaw signal**  $Y$  is:

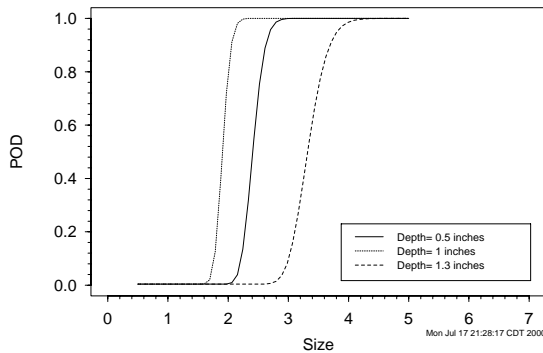
$$\Pr(Y \leq y; \underline{x}) = F_{y^{**}}(y; \underline{x}) = \Phi \left[ \frac{\Phi^{-1}(F_{Y^*}(y; \underline{x})) - \mu}{\sigma} \right]$$

- When  $\mu = 0$  and  $\sigma = 1$ ,  $\Pr(Y \leq y; \underline{x}) = F_{Y^*}(y; \underline{x}_i)$  is the same as the UT physical.
- Basic POD can be computed as

$$\begin{aligned} POD(a; \underline{x}) &= \Pr(Y > y_{\text{thresh}}) \\ &= 1 - \Pr(Y \leq y; \underline{x}) \end{aligned}$$

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### Model-Predicted POD as a Function of Size for Several Depths.



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### Concluding Remarks

- Physical models provide an effective methods for quantifying POD in situations when data on real flaws is limited.
- Statistical methods can be used to quantify and adjust for systematic biases and sources of variability that are not accounted for in the model.
- Improved computing capabilities and continued efforts in model development will increase the needs/opportunities for the use of models to quantify NDE inspection capability.

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