Fracture of soft elastic foam

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Abstract

Consisting of stretchable and flexible cell walls or ligaments, soft elastic foams exhibit extremely high fracture toughness. Using the analogy between the cellular structure and the network structure of rubbery polymers, this paper proposes a scaling law for the fracture energy of soft elastic foam. To verify the scaling law, a phase-field model for the fracture processes in soft elastic structures is developed. The numerical simulations in two-dimensional foam structures of various unit-cell geometries have all achieved good agreement with the scaling law. In addition, the dependences of the macroscopic fracture energy on geometric parameters such as the network connectivity and spatial orientation have also been revealed by the numerical results. To further enhance the fracture toughness, a type of soft foam structures with non-straight ligaments or folded cell walls has been proposed and its performance studied numerically. Simulations have shown that an effective fracture energy one order of magnitude higher than the base material can be reached by using the soft foam structure.

Keywords: elastic foam, elastomer, toughening

Introduction

Solid foam, a state of material characterized by the highly porous cellular structure, is commonly found in nature and in everyday life. In industrial applications, solid foams are well known for their superior energy-absorbing capability under compression [2-4]. Their fracture properties have also attracted great interests [4-9]. Scaling laws between fracture properties and porosity have been proposed and widely accepted [4,5]. However, most existing theories are based on linear elastic fracture mechanics and the cell walls of the foams are assumed to be linear elastic prior to rupture.
While such theories and predictions can be applied to foams of relatively stiff materials (e.g. ceramics and metals), their applicability becomes questionable to those consisting of soft and highly stretchable materials, such as elastomers. For example, an early experimental study on polyurethane foams found the fracture energy to be less dependent on the density, and even exhibiting a slight decrease when density increases [10], while the scaling laws of rigid foams all demonstrate linear or power-law dependence of the fracture energy over density [4,5].

The major difference between stiff brittle foam and soft elastic foam lies in the porosity and the slenderness of cell walls (or ligaments for an open cell foam). For simplicity, in the following discussion, the two types of foams will be referred to as rigid foam and soft foam, respectively. In contrast to the cell walls of rigid foam which partially shares the load after rupture, a fractured cell wall of soft foam merely dangles over the rest of the structure. Such a structural difference induces the dramatic distinction in energy transmission during fracture. Upon rupture, the remaining elastic energy in a cell wall of rigid foam could be redistributed according to the crack-tip advancement, while that in a slender component could hardly be transferred to its neighbors. The elastic energy in a ruptured slender component is mostly dissipated through local vibration or viscoelastic deformation. As a result, the effective fracture energy would have to include the energy of the entire component rather than just that at the vicinity of the crack faces (i.e. the surface energy).

To better understand this unique mechanism of toughening, one may consider the fracture process of a rubber, in which the crosslinked network of long polymer chains could be regarded as an extreme case of soft foam when each slender ligament shrinks down to a molecular scale. In their classic paper, Lake and Thomas suggest the fracture mechanism of rubber fracture: the energy needed to rupture a polymer chain is much larger than that of a single bond as the entire chain is subject to virtually the same breaking force [11]. After fracture, the broken chains recoil and their entropic elastic
energy would not be forwarded to neighboring chains. Thus, the intrinsic fracture energy of rubber scales approximately as \( \Gamma_{\text{rubber}} \sim N n \frac{1}{3} l_m U \), with \( N \) being the number of chains per unit volume, \( n \) the number of monomers per chain, \( l_m \) the length of each monomer, and \( U \) the energy needed to rupture each monomer [11]. This model has been widely used on rubber, but has seldom been related to the fracture of soft foam. Just by using the analogy, we may also deduce the scaling relation for the fracture energy of soft foam as

$$\Gamma \approx \alpha W_c \psi l \quad (1)$$

Here, we use \( W_c \) to represent the critical energy density of cell wall at rupture, \( \psi \) for the volume fraction (i.e. the relative density) of the solid phase, and \( l \) for the characteristic size of the foam (e.g. the height of a cell wall). \( \alpha \) is a dimensionless geometric factor. One may arrive at the same result from a different perspective. Due to the special geometry of foam, the sharpness of a crack is always limited by \( l \). The fracture energy is thus given by \( \Gamma \sim Wl \) [12], with \( W = \psi W_c \) being the effective strain energy density at the crack tip. In the limiting case when the material is stiff enough and the fracture process can be modeled by linear elastic fracture mechanics, by using the effective modulus of the structure \( E_{\text{eff}} \sim \psi^{\frac{2}{3}} \), this scaling law (1) reduces to the model of stiff foams with the fracture toughness given by \( K_{IC} \sim \sqrt{\Gamma E_{\text{eff}}} \sim \psi^{\frac{1}{3}} \sqrt{l} \) (for open cells) [4]. However, the applicability of the latter, which was derived from linear elastic fracture mechanics by assuming a square-root singularity in the stress field [4], to soft elastic foams undergoing large deformation remains unknown.

It is noteworthy that due to the presence of defects, the rupture strength and thus \( W_c \) is usually size dependent, unless the cell walls are thinner than the critical size for theoretical strength. The energy-dissipation mechanism of the soft foam structure, on the other hand, is never limited to
microscopic scale. One may refer to a two dimensional macroscopic analogy of soft foam, a net or a netted structure (e.g. a string bag), commonly known for its toughness and notch insensitiveness.

Even though the scaling law (1) seems natural and plausible, it could be hard to verify it directly through experiments. In practice, it is difficult to control the porosity and cell size independently during polymer processing, not to mention the size dependency of $W_\gamma$. Alternatively, this paper seeks to verify the scaling law through numerical modeling. In the following sections, a phase-field model for rubber fracture will be adopted to simulate the damage initiation and evolution in hyperelastic cellular structures. The phase-field model for fracture, which is capable of calculating the crack growth according to the energy criterion without a predetermined crack path, is very suitable for structures with complex geometries such as soft elastic foams. The scaling law and the special toughening mechanism will then be demonstrated with the phase-field model. The dependence on the detailed geometry of the foam cells will also be studied.

**Phase-field model of fracture**

Numerical simulation of fracture processes has the inherited difficulties in dealing with discontinuities, singularities, and moving boundaries which causes large geometric and even topologic changes. To overcome some of these difficulties, phase-field models of brittle fracture have been developed [1, 13-18, 22]. Recently, phase-field models have also been applied to the brittle fracture of rubbery polymers [23]. Numerical experiments have already shown that these models are capable of capturing both the onset of crack propagation and the damage morphologies of dynamic cracks [19-21]. Without the need to track individual crack or to prescribe a crack path, the phase field method becomes a promising candidate for modeling the fracture of structures with relatively complex geometries, such
as the soft elastic foam. The model used in this paper closely follows these developments, especially those by Karma et al [13] and Hakim and Karma [14].

To describe the state of material damage and to avoid tracking the crack front and faces, a phase field $\phi(\mathbf{X},t)$ varying continuously between the intact region ($\phi = 1$) and a fully damaged region ($\phi = 0$) is introduced. The loss of integrity in the solid is modeled by writing the elastic strain energy density as a monotonic increasing function of the damage variable, $g(\phi) W_s^0(\mathbf{F})$, where $W_s^0$ is the strain-energy density of the intact material under the same strain, and $g$ is an interpolation function in the interval $(0,1)$ with vanishing derivatives on both ends. In this study, we choose the interpolant $g(\phi) = 4\phi^3 - 3\phi^4$. As a common practice of hyperelasticity, the deformation gradient tensor $\mathbf{F}$ is used to represent the state of strain. Following Karma et al. [13], we write the free energy density function to include three contributing terms:

$$W(\phi, \nabla \phi, \mathbf{F}) = g(\phi) W_s^0(\mathbf{F}) + [1 - g(\phi)] W_s + \frac{\kappa}{2} \| \nabla \phi \|^2. \quad (2)$$

The second term on the right hand side of Eq. (2) represents the energy associated with material damage. When the strain energy at a material particle exceeds the threshold $W_s$, the damaged state with $\phi = 0$ becomes energetically favorable. Just as in almost all phase-field models, the gradient energy term is added to regulate a smooth transition between the coexisting states. In this paper, the material constituting the solid phase of the foam is assumed to be isotropic, so that only a scalar coefficient $\kappa$ is needed for the gradient energy term. In equilibrium, the combination of the second and third terms on the right hand side of Eq. (2) gives the surface energy, i.e. half of the intrinsic fracture energy. The fracture energy of the material modeled by the energy function is approximately
\[ \Gamma \approx 2 \sqrt{kW_e} \] [13]. Here, in a body undergoing finite deformation, all energy densities are measured with respect to the volume in the reference state.

Countless number of constitutive models have been developed for hyperelastic solids. Although specific stress-strain relations of the solid phase may affect the ultimate fracture properties of soft foam, such dependence is beyond the scope of the current paper. Here for simplicity, we will limit the discussion to a neo-Hookean material of the strain energy function

\[ W_e^0(F) = \frac{\mu}{2} (F : F - 3), \] (3)

where \( \mu \) is the initial shear modulus. In contrast to linear elastic solids, rubbery polymers are often modelled as incompressible. To enforce volume incompressibility, approaches such as the application of the Lagrange multiplier are often taken, e.g. by adding to the free energy function a term \(- p(\det F - 1)\) with a Lagrange multiplier \( p \) representing the pressure field. A physically meaningful model needs to degrade the compressibility simultaneously with the shear stiffness. Directly multiplying the Lagrange multiplier term by \( g(\phi) \) obviously does not serve the purpose. Instead, we modify the Lagrange multiplier term slightly by modeling the material as slightly compressible:

\[ - p g(\phi)(\det F - 1) - \frac{1}{2} \frac{p^2}{K}. \] (4)

By taking the variation of (4) with respect to \( p \), one will arrive at an equation of state with a degrading bulk modulus: \( p = - g(\phi)K(\det F - 1) \). In the intact state, the large bulk modulus \( K \gg \mu \) ensures volume conservation; in the fully damaged state, the added term does not affect the field of deformation, and the ad-hoc field \( p \) is regulated numerically by the quadratic term in (4).
With all the aforementioned energy contributions, the total free energy of the system is simply the volume integral of the energy density, including the terms in (4), and the surface integral of the potential of external tractions \( t \):

\[
\Pi[\phi, \mathbf{x}] = \int W dV - \int \mathbf{t} \cdot dA ,
\]

where \( \mathbf{x}(\mathbf{X}, t) \) symbolizes the current coordinates of a material particle located at \( \mathbf{X} \) in the reference state. The total free energy \( \Pi \) is a functional of the field of damage \( \phi(\mathbf{X}, t) \) and the field of deformation characterized by \( \mathbf{x}(\mathbf{X}, t) \). Following Hakim and Karma [14], we neglect inertia and body forces, and assume the system to be in partial mechanical equilibrium, so that \( \partial \Pi / \partial \mathbf{x} = 0 \) or

\[
\nabla \cdot \mathbf{s} = 0 ,
\]

in the bulk, and \( \mathbf{N} \cdot \mathbf{s} = \mathbf{t} \) on the surfaces. Here \( \mathbf{s} = \partial W / \partial \mathbf{F} \) is the nominal stress, and \( \mathbf{N} \) is the unit normal vector on a surface. For the evolution of the phase field \( \phi \), on the other hand, we assume a linear kinetic law with isotropic mobility \( m \):

\[
\dot{\phi} = -m \frac{\partial \Pi}{\partial \phi} = m [\kappa \nabla^2 \phi - g'(\phi) (W^0_x + p \det \mathbf{F} - W_c)] .
\]

To simulate quasi-static fracture processes, a large enough mobility is taken to achieve rate-independent results. Further, to model the irreversibility of fracture processes and to prevent the damaged phase from healing, we force \( \phi \) to be a monotonically decreasing function of time by taking only the positive part of the driving force [22]:

\[
\dot{\phi} = -m \left( g'(\phi) (W^0_x + p \det \mathbf{F} - W_c) - \kappa \nabla^2 \phi \right) ,
\]
where the angular brackets indicate an operation of taking the positive values, \( \langle \xi \rangle = (\xi + |\xi|)/2 \).

Supplemented by proper initial and boundary conditions, Eqs. (6) and (8) constitute a partial differential system for the coevolution of deformation and damage fields, \( \mathbf{x}(\mathbf{X}, t) \) and \( \phi(\mathbf{X}, t) \).

![Diagram](image)

**Fig. 1.** The rupture and retraction process of a ligament (or cell wall) in a foam structure. Because of its slenderness, a soft filament will tend to buckle or coil and could not effectively transduce energy.

**Energy dissipation and numerical implementation**

The major difference between soft and stiff foams and the primary means of energy dissipation during fracture can be qualitatively understood with the aid of Fig. 1. For ease of description, we will refer to the solid dividing segments in both open and closed foams as *ligaments* from now on. Upon rupture from stretched states, a ligament will first be accelerated by its own retracting forces, and the elastic energy stored prior to rupture is mostly converted to kinetic energy. When the ligament retracts further, the difference between stiff and soft ligaments is revealed: while a stiff ligament will remain straight and decelerate and transfer the energy further to the neighboring components, a soft ligament will tend to buckle or coil due to its slenderness, and the energy could not be effectively transferred. The ultimate factor is the stiffness ratio between the surrounding structure and the broken ligament (a buckled ligament has very low stiffness), as elastic wave cannot propagate from a compliant medium to
a rigid one. As a result, the elastic energy of a soft ligament is mostly damped through subsequent vibration of itself.

The detailed process and the dependence on the structural geometry, such as the aspect ratio of each ligament and the spatial connectivity at each node, can be simulated by computing the full dynamic response of a ligament and the surrounding structure. Such analysis, however, is not of particular interest to the current paper. We will focus on soft foams with very slender ligaments, and hypothesis that most part of the elastic energy stored in the broken ligaments will be dissipated through this process. The results are thus inapplicable to relatively stiff foams. On the other hand, it is also computationally less feasible to model the full dynamic behavior of each ligament in a foam structure of complex geometry. Instead, we will neglect the inertia and model the fracture process as quasi-static. In this limit, the dissipation through crack propagation is negligible, and the fracture energy is mainly dissipated through viscosity. Without considering dynamics, the snap back of the ligaments are fully damped after each rupture event. Instead of a proof or evidence, the calculations presented as follows are the consequence of the proposed energy-dissipation mechanism.

Similar as in many other methods for fracture and damage simulation (e.g. cohesive element and element deletion), without inertia, the damage-induced softening is intrinsically unstable. To stabilize numerical procedures, and more importantly to dissipate the redundant strain energy in the dangling ligaments after rupture, we introduce a Newtonian-fluid-like damping term to the nominal stress

$$s = \frac{\partial W}{\partial F} + \frac{\eta}{2} \nabla \dot{x} \tag{9}$$

with $\eta$ being the numerical viscosity. For relatively small viscosity $\eta$, the viscous stress only has significant contribution at the regions of high deformation rate, which is expected to occur only in a
retracting ligament upon rupture. Numerical experiments have shown that when a small value of $\eta$ is taken, the artificial viscosity only changes the rate of structural unloading at the wake of a propagating crack, and does not affect the energy consumption.

Substituting Eq. (9) into (6), one may obtain the coevolution of deformation and damage fields by solving the partial differential system (6) and (8) simultaneously. The system has an intrinsic length scale, $r = \sqrt{k/W_c}$, which is approximately the thickness of the transition zone from the intact region to a full damaged region. It could be argued that $r$ physically characterizes the width of the fracture process zone in the condensed solid phase. Without losing generality, we rewrite the governing equations into a dimensionless form by normalizing all energy densities and stresses by $W_c$, all lengths by $r$, and time by $1/mW_c$. After normalization, the dimensionless fracture energy of the solid phase is approximately 2, and the system has only three dimensionless parameters: the normalized shear modulus $\mu/W_c$, bulk modulus $K/W_c$, and viscosity $m\eta$. In the following numerical examples, we will take dimensionless modulus $\mu/W_c = 0.2$, which corresponds to the representative values of a soft elastomer: $\mu \sim 10$ MPa, $r \sim 1\mu$m, and $\Gamma \sim 50$ N/m. The dimensionless bulk modulus is taken to be $K/W_c = 200$, and the viscosity $m\eta = 10^{-4}$. It should also be noted that the ligament thicknesses of most structures calculated in the current paper are comparable to the intrinsic length scale $r$. In this limit, the calculations could as well be done by using the regular strength-based material degradation approach. Here, the phase-field approach is taken so that the direct comparison with the fracture process of a bulk material with the same property could be made when needed.

The dimensionless equations are implemented into a finite-element code through the commercial software COMSOL Multiphysics 4.3b. For numerical robustness under large deformation, the geometries are discretized by using triangular elements, and both the displacement and damage
fields are interpolated with linear Lagrange shape functions. To capture the transition at the interface between the damaged and intact phases, a maximum mesh size of $0.5R$ is prescribed. The model is integrated over time via a fully coupled implicit scheme, with adaptive step size. To numerically enable damage nucleation, spatial random distributions of the initial shear modulus and the intrinsic fracture energy has been introduced to each model, with standard deviation at 1% of the corresponding magnitudes.

![Fig. 2. Sketch of the loading conditions for the foam structures](image)

To compute the fracture energy, we load the pre-cracked structures in a similar way as the pure-shear test for rubber. As sketched in Fig. 2, the right and bottom edges are constrained by rollers, and the top edge is loaded by a uniform displacement. For symmetric structures and if the crack propagates along a symmetry line, only half of the structure is calculated and a symmetry boundary condition is prescribed along the symmetry line. A ramping displacement load is applied within a short time and then held constant. The crack will start to propagate when the applied displacement exceeds certain value. In a steady state when the crack tip is far from either ends, the energy release rate is independent of the crack length,

$$ G = W_{at} H, $$  \hspace{1cm} (10)
where $H$ is the undeformed height of the structure. $W_{\text{eff}}$ is the effective strain-energy density in the absence of the crack, and is averaged over the volume including the space of the pores. In contrast to the standard pure-shear test, the entire structure is under plane-strain condition, and is allowed to shrink horizontally. The corresponding 2D results are closer to the behavior of 3D closed foam. Although the effective strain-energy density $W_{\text{eff}}$ can be calculated by integrating $W$ at the region far ahead of the crack tip, here we calculate it separately by subjecting a non-cracked structure to plane-strain uniaxial tension.

**Results and discussion**

The simulations are first carried out on hexagonal (honeycomb) structures, as shown schematically in Fig. 3a. In order to reduce boundary effect, the actual computational domain is much larger than that shown. Fillets of dimensionless radius 1 have been applied to all corners to reduce stress concentration, as the preferential damage of the triple junctions may result in a different scaling law. The local deformation and damage fields of a representative result are shown in Fig. 3b, in which the preexisting crack has propagated through three ligaments.
Fig 3. (a) Part of the 2D honeycomb foam structure being simulated. (b) The calculated deformation and damage fields of a honeycomb structure, during the propagation of a preexisting crack. The shades represent the dimensionless strain energy density $W/W_c$. The crack profile is indicated schematically by the dash line, which goes through the transition zone from the intact to the fully damaged regions in terms of $\phi$. The deformation is shown to scale, and only part of the structure near the crack tip is shown. The actual computational domain is much larger than that shown to circumvent size-effect.

In a steady state, a crack is propagating through the structure at a constant speed, the energy release rate $G$ is given by Eq. (10). However, due to the discrete nature of the structure, the crack propagation appears staggered. To capture the effective crack velocity, we identify each event of ligament rupture, and record the time of the event and the horizontal coordinate of the corresponding ligament in the undeformed state, as shown by Fig. 4a. It is found that when the crack front is far from the edge, the ligament-rupture events are almost equally distributed in time, indicating a steady-state crack propagation. The slope of the linear fit to the rupture events under each loading condition is taken to be the nominal crack velocity $v$. In consequence of the kinetic law, Eq. (7) or (8), the energy release rate $G$, i.e. the driving force of the crack, is rate dependent. As shown by Fig. 4b, the effective fracture energy of the structure is a monotonic increasing function of the crack velocity. Here, to compare between different structures, we use the threshold value $\Gamma$ calculated from the vertical intercept of the fracture-energy-velocity curve, as shown by Fig. 4b. The threshold corresponds to the fracture energy of a crack propagating quasi-statically at zero velocity. Following such a procedure, we compute the quasi-static fracture energy of various foam structures, presented as follows.
Fig. 4. (a) Undeformed coordinates of the ligaments as a function of the times of rupture. The line is the best linear fit. The slope indicates the dimensionless crack velocity. (b) Dimensionless fracture energy (energy-release rate) \( \Gamma/\ell W_c \) as a function of the dimensionless crack velocity \( \nu/\ell W_c \). The line is the best linear fit, and the vertical intercept shows the quasi-static fracture energy of the soft foam.

The scaling relation (1) is verified first through the simulation on the fracture processes of hexagonal soft foams. A set of two-dimensional hexagonal foams of the same volume fraction \( \varphi = 0.09 \) but different ligament lengths are modelled, and their quasi-static fracture energies are computed via the same procedure as described above. The resulting fracture energies of two different orientations are plotted against the ligament length in Fig. 5a. In both cases, the dimensionless fracture energy \( \Gamma/\ell W_c \) is approximately linear in the ligament length \( \ell/\ell \). Similarly, we fix the length of each ligament at \( \ell/\ell = 4.2 \), and vary the solid volume fraction from \( \varphi = 0.025 \) to \( \varphi = 0.17 \). The resulting fracture energies of the two orientations are plotted as functions of the solid volume fraction in Fig. 5b. As expected, at relatively small volume fraction, the fracture energy is approximately proportional to the volume fraction \( \varphi \).

Comparing between the two orientations, it is found that the fracture energy in an “armchair” orientation is consistently higher than that of the same structure in a “zigzag” orientation. Such a
difference could be attributed to the anisotropy in ligament density. As illustrated by Fig. 6, a horizontal crack mainly goes through the inclined ligaments in the “armchair” orientation, while a crack through a foam in the “zigzag” orientation mainly breaks the vertical ligaments. The numbers of ligaments cut by unit crack length in the two orientations differ by a factor of $2\sqrt{3}$, which explains the difference in the effective fracture energies. The same phenomena may also be understood by considering the effective sharpness of a crack. As shown by Fig. 6, the crack path in a “zigzag” orientation is nearly straight, while that in an “armchair” orientation is often meandering. With the crack front randomly selects one of two inclined ligaments, which has almost identical strain energies, the effective crack tip can be regarded as encompassing the region of both ligaments, and thus the crack is blunter.

Fig. 5. Calculated fracture energy of the hexagonal soft foams versus (a) the normalized ligament length $l/r$ at constant volume fraction $\psi = 9\%$, and (b) the volume fraction of the solid phase $\psi$ at constant ligament length $l/r = 4.2$. Two different orientations are simulated as indicated by the insets (with horizontal cracks).
Fig. 6. Damage patterns of hexagonal soft foams in (a) “zigzag” and (b) “armchair” orientations. The color scale represents the damage variable $\phi$, plotted in the undeformed geometry. The dash curves show the approximate paths of crack propagation.

The simulations on the crack propagation processes in soft foam structures of various geometries, including those with triangular and square unit cells at different orientations, all exhibit the similar linearity as that observed in the honeycomb foam, which further supports the scaling relation (1). The effect of unit-cell geometry is only reflected in the dimensionless coefficient $\alpha$, as summarized by Fig. 7. Although the relatively high fracture energy of the square-cell foam with vertical/horizontal ligaments may be explained by the higher ligament density than that of the hexagonal foam (one ligament per crack length $l$ versus one ligament per $\sqrt{3}l$ or $2l$), the foams of other patterns do not follow the same trend. Despite the higher ligament densities, the effective fracture energies of the triangular foam or the rotated square foam are actually lower. To understand the relatively low fracture energies, let us revisit the physical origin of the polymer-network-like toughening mechanism. Two necessary conditions must be met for the mechanism to be effective: (a) the ligaments must be relatively long and uniform, so that the elastic strain energy everywhere along a ligament is close to critical prior to rupture; (b) the network structure must be sufficiently compliant, so that the remaining strain energy after rupture is not passed to the neighboring ligaments. Even with the same aspect ratio and uniformity in the ligaments, the network connectivity in either the triangular foam or the square-
cell foam is higher than that in the hexagon foam. Each node is connected to six ligaments in a triangular foam, and four in a square-cell foam, but only three ligaments are connected to each node in a hexagonal foam. Therefore, at the tip of a propagating crack in a triangular foam of square-cell foam, two or more ligaments connected to the same node will be stretched and almost aligned in the direction perpendicular to the crack. Once one of them ruptures, the remaining elastic strain energy may be partially transferred through the common node to the other ligaments which are still standing and carrying the load along the same direction. The strain energy carried over may contribute to the further propagation of the crack, and the overall fracture energy is thus lower. It should be noted that the square-cell foam with vertical and horizontal ligaments represents a special case, in which the two lateral ligaments are connected at the direction almost perpendicular to the load, and thus the macroscopic fracture energy is less affected by the relatively high connectivity.

Fig.7. Geometric effect on the fracture energy of soft foams. The geometries and orientations are represented by the sketch insets, with cracks running horizontally.

Despite the apparent linear relation between the effective fracture energy and the solid volume fraction $\psi$ (or the ligament length $l$), the polymer-network-like toughening mechanism is unlikely to make a soft cellular material with macroscopic pores tougher than the bulk solid. By using the same
method on a bulk solid, we have confirmed that the dimensionless fracture energy of a condensed structure is approximately 2, just as shown in the literature [23]. The factor which has not been taken into consideration here is the size-dependency of material strength. It is well-known that, due to the presence of defects, larger samples of the same material would exhibit a lower tensile strength. The critical energy density $W_c$ which scales with the squared of the rupture strength, is dependent on the ligament thickness $d$ (and is usually a decreasing function). On the other hand, without resolving microscopic defects, the phase-field model used in the current paper will not predict any size effect, and $W_c$ is taken as a material parameter for normalization. Instead, if we assume the scaling relation of brittle solids from linear elastic fracture mechanics $W_c \sim d^{-2n}$ with the scaling index $n = 1/2$ [25], and follow the geometric relation for regular closed-cell foams $d \sim \psi l$, we will arrive at an effective fracture energy almost independent of $\psi$ or $l$. For non-brittle materials, the scaling index $n$ is usually less than $1/2$, and the effective fracture energy will be weakly dependent on $\psi$ and $l$. Furthermore, in the limiting case when the ligaments are thin enough that the theoretical strength could be achieved, $W_c$ will become size-independent, and the scaling law (1) could be fully recovered. The size-dependency of tensile strength, which has been extensively studied [24], is not a main focus of the current paper.

Here we further investigate geometric effects by studying soft elastic foams with non-straight ligaments. The scaling relation, $d \sim \psi l$, represents the geometry of closed-cell foams with relatively straight ligaments (or flat cell walls). In general, if one allows non-straight or folded ligaments, the volume fraction $\psi$ can be varied independently from the ligament thickness $d$. In other words, one may increase the solid volume fraction $\psi$ while keeping the ligament thickness small to achieve higher fracture toughness in the structure.
Fig. 8. (a) Sketch of the unit cell of a soft elastic foam containing serpentine ligaments. (b) Simulated deformation and fracture process in the soft elastic foam. The shading shows the dimensionless strain energy density $W/W_r$.

The deformed shape is plotted by downscaling the actual displacement value to 10%.

As an illustrative example, we construct a numerical model by repeating the unit cell as sketched in Fig. 8 (a). Unlike in the above examples, the initial geometry of a ligament takes a serpentine form. The material is taken to be soft enough so that the ligaments are insensitive to the stress concentration at the folding corners. During deformation, the ligaments will first be straightened and then rupture. As shown by Fig. 8b, the strain-energy distribution in the ligaments at the crack tip is still close to uniform, with the value close to $W_r$ prior to rupture. By varying the width of the serpentine pattern and using the same method as in previous examples, we evaluate the fracture energy of several structures with ligament thickness taken to be $r$ and the unit cell size $18.8r \times 18.8r$, and plot it as a function of the solid volume fraction. As shown by Fig. 9, the scaling law (1) still holds for the foam structures with serpentine ligaments. Despite the different geometries of the unit cells, the extrapolation of the curve to lower volume fractions will give similar fracture energy levels as structures with straight ligaments. However, due to the much more condensed nature of the folding structures, the fracture energy is significantly improved. For the structure shown by Fig. 8, the volume fraction
reaches $\psi \approx 75\%$, and the dimensionless fracture energy is $\Gamma/\psi W_c \approx 25$, more than one order of magnitude higher than the foam structures with straight ligaments or the same material in a bulk form.

![Figure 9](image.png)

**Fig. 9.** Calculated fracture energy of soft foam structures with serpentine ligaments, as a function of the solid volume fraction $\psi$. The structures have identical unit-cell size and ligament thickness. The volume fraction is controlled by changing the width of the serpentine pattern, as indicated by the insets.

Without realistic material models or optimized design parameters, this numerical example is just an illustration of the toughening mechanism. Nevertheless, it is evident that the fracture energy of a material may be significantly increased by adopting similar structural designs. More interestingly, such a toughening mechanism is not just limited to soft solids, especially when the ligament size is small. It should also be noted that the fracture energy increase is obtained at the expense of the initial structural stiffness. As shown by Fig. 10., the effective initial modulus is more than two orders of magnitude lower than that of the constituting solid. Such a relation between structural compliance and fracture toughness is similar to that in the microcrack-toughening mechanism, although the latter is usually studied in the context of linear elastic fracture mechanics [26-28].
Fig. 10. The calculated nominal-stress-stretch curve of a soft foam with serpentine ligaments as shown by Fig. 8 (without a pre-existing crack). The initial stiffness of the structure is more than two orders of magnitude lower than the solid material. The stress-stretch curve exhibits a strain-stiffening behavior, even though the material is taken to be neo-Hookean.

Moreover, even though the material is taken to be neo-Hookean, the stress-stretch curve exhibit a clear strain-stiffening segment at relatively large stretch, just like the behavior of elastomers at the stretch limit of polymer chains. Here, the strain stiffening corresponds to the straightening of the serpentine ligaments. To some extent, such a structure can be regarded as a macroscopic model system for the fracture of elastomers.

The detailed design and optimization of structures of the kind, although interesting, is beyond the scope of the current paper. We are eagerly awaiting the designs and manufacturing of tough materials by utilizing this mechanism.

Conclusion

Drawing an analogy between the compliant ligaments in a soft elastic foam and the polymer chains in an elastomer, this paper proposes a polymer-network-like toughening mechanism and derives
the scaling relation between the macroscopic fracture energy and the structural characteristics of soft foam structures. Different from the energy absorbing mechanism of rigid foams which is mainly effective at compression, the polymer-network-like toughening mechanism allows a soft foam to effectively dissipate energy when the structure is subject to tension. Through a phase-field model developed specifically for the fracture of elastomers, the toughening mechanism as well as the scaling relation is then verified on soft foam structures of various geometries. In addition to the scaling law, it is found that the geometric parameters such as the ligament density and the network connectivity will also affect the fracture energy of soft foams. Finally, to increase the volume fraction of the solid phase without affecting the thickness or slenderness of each ligament, a type of soft foam structures of serpentine ligaments is proposed. Numerical study suggests that such structures may reach an effective fracture energy much higher than that of the corresponding bulk material. In other words, one may toughen a soft material just by cutting slots or holes in it.

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References


