Magnetostriction and Field Stiffening of Magneto-Active Elastomers

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Filled with certain amount of magnetic particles, an elastomer can be made magneto active for numerous applications. When a magneto-active elastomer (MAE) is subject to a homogeneous magnetic field, both magnetostriction and field-stiffening effect can be observed. Inspired by experimental observations and microstructure simulations in the literature, this paper presents a simplified phenomenological model for MAEs by considering a uniaxial deformation state. The model hypothesizes the field-stiffening effect to be a direct consequence of the inverse magnetostriction, i.e., the strain-dependent magnetization, in the context of finite deformation. By taking the elastic energy to be independent of magnetic field and the magnetization energy to be strain dependent, the model can capture both magnetostriction and field stiffening of an MAE. The functional form of the strain-dependent magnetization energy is determined by the underlying microstructure. MAEs with different microstructures exhibit different magnetostriction and field-stiffening behaviors. To predict the behavior of a specific MAE, one only needs to measure the effective permeability of an MAE as a function of the axial strain. The mathematical simplicity of the model could enable simulation and optimization of MAE-based devices under complex loading conditions.

Keywords: Magneto-active polymer; magnetostriction; magneto-rheological elastomer.

1. Introduction

Although polymers are usually nonmagnetic, they can be made magneto-active by incorporating ferromagnetic filler particles. In this paper, we will focus on polymer matrices exhibiting rubbery behavior with negligible viscosity or plasticity, and we will refer to the resulting composites as magneto-active elastomers (MAEs). As a
type of soft active materials, MAEs possess two important features. In response to an applied magnetic field (uniform or non-uniform), it may deform as well as change its apparent stiffness. The deformation induced by the applied magnetic field, especially a uniform field, is commonly known as magnetostriction. While magnetostriction is a universal phenomenon for almost all ferromagnetic materials [Joule, 1842; Clark and Belson, 1983; Wun-Fogle et al., 1999], it is much more pronounced in MAEs due to their soft nature [Bednarek, 1999; Ginder et al., 2002; Martin et al., 2006; Guan et al., 2008]. For example, an axial strain of 1.5% can be achieved from a silicone rubber filled with iron particles [Coquelle and Bossis, 2005]. Even larger magnetostriction was observed on MAEs with porous polymer matrices [Bednarek, 2006]. The large magnetostriction makes MAEs suitable for a variety of active devices, such as transducers, actuators, and sensors [Kim et al., 2011; Böse et al., 2012; Chan et al., 2012]. The second significant feature of MAEs, which will be referred to as the magneto-rheological (MR) effect hereafter, is their changeable stiffness by a magnetic field. Increases in the shear and tensile moduli under a magnetic field have both been observed in MAEs [Jolly et al., 1996; Gong et al., 2005; Varga et al., 2006]. Unlike magnetostriction, the MR effect is rarely observed or studied in conventional ferroelectric solids. Such MR effect of MAEs enables their applications in which a tunable stiffness is required, such as smart vibration dampers [Deng et al., 2006; Li et al., 2012], active vibration controls for earthquake [Ni et al., 2010], tunable noise barriers [Farshad and Roux, 2004], and cellular therapies [Chan et al., 2012].

To facilitate utilization of these soft active materials, experimental and theoretical studies have been continuously conducted. It has been found that both magnetostriction and MR effect are highly dependent on the underlying microstructures. Conventionally, MAEs are divided into two main categories by the particle distribution. An isotropic MAE, cured in the absence of a magnetic field, has filler particles randomly dispersed in the polymer matrix [Zrínyi et al., 1997; Faidley et al., 2010]. An anisotropic MAE, on the other hand, is cured under a magnetic field, thence the iron particles tend to be aligned along the direction of the applied field [Jolly et al., 1996; Gong et al., 2005; Varga et al., 2006]. Experiments showed that the magnetostriction in isotropic MAEs can be one order of magnitude higher than that in anisotropic samples [Guan et al., 2008; Danas et al., 2012]. Besides, it is found that under a relatively low magnetic field, the magnitude of magnetostrictive strain is approximately proportional to the squared of the field strength, while under a relatively high field the magnetostriction tends to saturate [Ginder et al., 2002; Martin et al., 2006]. Several models based on magnetic dipole interactions have been developed to explain the magnetostriction in isotropic MAEs [Digué et al., 2009; Stolbov et al., 2011]. However, very few models work for both isotropic and anisotropic MAEs. While experiments show elongation in an anisotropic MAE, some theories predict that the dipolar attraction between neighboring particles in a chain leads to shortening [Kankanala and Triantaffylidis, 2004].
Similarly, the origin of the MR effect has also been attributed to the attractions between adjacent filler particles in the material [Jolly et al., 1996; Davis, 1999]. However, by assuming the filler particles to be aligned in straight chains, many models fail to explain the MR effect in the tensile modulus [Ivaneyko et al., 2011]. Recently, a microstructure-based model points out that the zigzag nature of particle chains is the key to the MR effect in both shear and tension [Han et al., 2013]. Although all these models reveal the importance of the microstructures, it is practically impossible to model the geometric details from the actual structure of an MAE to the particle level, especially when predicting the performance of an MAE-based device under complex loading conditions. Hence, a phenomenological model which treats the composite as a homogeneous continuum is preferred in engineering applications.

Other than magnetostriction and MR effect, the inverse magnetostriction (also known as the Villari effect) of MAEs [Bozorth, 1993], namely the change in the magnetic permeability in response to a mechanical deformation, has not attracted much attention. Although homogenizing rules such as the Maxwell–Garnett formula and the Bruggeman formula have been used to estimate the effective permeability of magnetic composites [Waki et al., 2006; Zhang et al., 2008], the strain dependence of permeability and its relationship to the magneto-mechanical properties of MAEs has seldom been studied. As the reciprocal effect of magnetostriction, inverse magnetostriction is naturally related to magnetostriction and MR effect. In fact, one may get the dominating coupling behavior of an MAE simply by looking at its inverse magnetostrictive property. Based on an assumption that the elastic energy is magnetic-field-independent while the magnetization energy is a function of the mechanical strain, the current paper proposes a simple homogeneous model capable of predicting both the magnetostriction and the MR effect of different types of MAEs.

2. The Model

The central hypothesis of the model could be understood from an energy point of view. The MAEs under consideration are composite materials consisting of a non-magnetic polymer matrix and dispersed magnetic filler particles. The magnetostriction of filler particles is negligible compared with the overall deformation of the composite. The total free-energy density of an MAE is commonly written into the sum of the elastic contribution \( W_e \) and the magnetic contribution \( W_m \) [Kankanala and Triantafyllidis, 2004; Dorfmann and Ogden, 2004]:

\[
W = W_e + W_m.
\]  

(1)

Because the filler particles are often orders-of-magnitude stiffer than the elastomer matrix, the elastic energy is mainly stored in the latter. Due to the high permeability of the filler particles, the actual deformation of the matrix is nonaffine: Particle
chains could slightly reconfigure under a magnetic field even when the overall deformation is constrained. However, such localized heterogeneity has little contribution to the total elastic energy or to the magneto-mechanical coupling [Han et al., 2013]. We therefore neglect this contribution and assume the deformation to be affine and the elastic energy to be independent of magnetic field: \( W_e = W_e(\lambda_1, \lambda_2, \lambda_3) \), where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the principal stretches in orthogonal directions. The magnetization, on the other hand, is highly dependent on strain. This is because the elastic deformation changes the relative position of filler particles and thus the effective permeability of the composite. As an example, imagine a chain of iron particles embedded in the elastomer substrate. The iron particles are much stiffer than the substrate, and magnetically much more permeable than the latter. If the chain is straight, an axial compression brings the particles closer and increases the effective permeability, as illustrated in Fig. 1(a). For a zig-zag chain, as in Fig. 1(b), an axial compression increases the angle between the more permeable pathway and the applied field, and thus the deformation may decrease the permeability. In the following discussion, the theory will be exemplified through specific energy functions, some of which identical to existing models. However, the essence of the theory is that the coupling behaviors of typical MAEs are dominantly governed by the strain-dependent magnetization: \( W_m \) is a function of strain, but \( W_e \) is independent of the magnetic field. From this starting point, various models can be constructed and tested through experiments.

For simplicity, in this paper we only consider a commonly encountered case when the material is under uniaxial tension or compression, and the magnetic field is also applied in the axial direction. The undeformed state in the absence of magnetic field is chosen to be the reference state. Let \( \lambda_1 = \lambda \) be the axial stretch. The volumetric incompressibility gives rise to the lateral stretches \( \lambda_2 = \lambda_3 = 1/\sqrt{\lambda}. \) As \( \lambda \) is the only deformation measure in this case, we can write the free-energy density as

\[
W(\lambda, \tilde{B}) = W_e(\lambda) + W_m(\lambda, \tilde{B}).
\]

Fig. 1. Schematic of particle rearrangement and the resulting strain dependence of magnetization in MAEs. (a) When a straight chain is under compression, the particle–particle distance reduces and the MAE becomes more permeable. (b) When compressed, a nonstraight chain becomes more jagged and may reduce the overall permeability of the MAE.
To account for the effect of large deformation, we differentiate the physical quantities defined in the reference configuration (the nominal quantities) from those defined in the deformed configuration (the true quantities). Here, $\tilde{B}$ is the nominal magnetic flux density, namely the magnetic flux going through a material surface of unit area at the reference state. It is related to the true flux density $B$ (flux per unit deformed area) by $\lambda \tilde{B} = B$ [Dorfmann and Ogden, 2004]. The free-energy functions are also defined as the energy stored per unit reference volume. Under such a Lagrangian framework, the nominal axial stress is given by [Dorfmann and Ogden, 2004]

$$s = \frac{\partial W(\lambda, \tilde{B})}{\partial \lambda},$$

(3)

and the nominal magnetic field

$$\tilde{H} = \frac{\partial W(\lambda, \tilde{B})}{\partial \tilde{B}} = \frac{\partial W_m(\lambda, \tilde{B})}{\partial \tilde{B}}.$$  

(4)

$\tilde{H}$ is defined as the surface current per unit reference length, and is related to the true counterpart $H$ as $\tilde{H} = \lambda H$ [Dorfmann and Ogden, 2004]. When the functional form of $W(\lambda, \tilde{B})$ is specified, Eqs. (3) and (4) become the equations of state of the particular material. Due to incompressibility, an arbitrary hydrostatic stress could be added to the material without altering its thermodynamic state.

To ease mathematical representation, we introduce another free-energy function by using the Legendre transform $\hat{W}(\lambda, \tilde{H}) = W(\lambda, \tilde{B}) - \tilde{B} \tilde{H}$. The transformed magnetization energy $\hat{W}_m$ is also a function of the nominal magnetic field: $\hat{W}_m(\lambda, \tilde{H}) = -W_m(\lambda, \tilde{B}) + \tilde{B} \tilde{H}$. The equations of state now read:

$$s = \frac{\partial \hat{W}(\lambda, \tilde{H})}{\partial \lambda} \quad \text{and} \quad \tilde{B} = \frac{\partial \hat{W}_m(\lambda, \tilde{H})}{\partial \tilde{H}}.$$

(5)

For finite but moderate deformation, a good approximation of the elastic energy could be achieved by the neo-Hookean model [Flory, 1953]:

$$W_e(\lambda) = \frac{G}{2} \left( \lambda^2 + \frac{2}{\lambda} - 3 \right),$$

(6)

where $G$ is the initial shear modulus of the composite. To describe the magnetization property of MAEs [Ponte Castañeda and Galipeau, 2011; Bica, 2012; Galipeau and Ponte Castañeda, 2013], the Langevin function has often been used, similar as in the models of conventional ferromagnetic materials. As detailed in Sec. 3.2, our experimental results of the magnetization-magnetic-field relation also fit well to the Langevin function. Therefore, we assume the following form of the modified magnetization energy:

$$\hat{W}_m(\lambda, \tilde{H}) = \frac{1}{2} \mu_0 H^2 + \frac{\mu_0}{\alpha} M_0(\lambda) \ln \frac{\sinh \alpha H}{\alpha H},$$

(7)

where $\mu_0$ is the vacuum permeability, $\alpha$ is a parameter characterizing the saturation field, and $M_0$ is the saturation magnetization. For most ferromagnetic materials, saturation is reached when the magnetizations in all microscopic domains are aligned.
to the applied field, and the saturation magnetization is thus considered constant. However, as illustrated by Fig. 1, it is not the case for MAEs, in which the magnetically aligned particles can still distribute differently in space. In general, both $\alpha$ and $M_0$ could be strain dependent. Here for simplicity, we have assumed that only the saturation magnetization is a function of stretch $M_0(\lambda)$. In the high-field limit, although the self-energy of the filler particles (namely the Zeeman energy) is much greater than the dipolar interaction energy, the strain-dependence of the latter should not be neglected, as the former is strain-independent. As shown by our experimental results and analyses in Sec. 3.2, the weak dependence $M_0(\lambda)$ actually induces significant magnetostrictive and magneto-rheological effects under high magnetic fields.

Substituting the geometric relation $\tilde{H} = \lambda H$ into Eq. (7) and the result further into (4), together with Eq. (6), we obtain the nominal stress

$$s = G \left( \lambda - \frac{1}{\lambda^2} \right) + \mu_0 \frac{H^2}{\lambda} + \mu_0 M_0 \frac{H}{\lambda} \left( \coth \alpha H - \frac{1}{\alpha H} \right),$$

and nominal flux density

$$\tilde{B} = \mu_0 \frac{\tilde{H}}{\lambda^2} + \mu_0 M_0 \frac{1}{\lambda} \left( \coth \frac{\alpha \tilde{H}}{\lambda} - \frac{\lambda}{\alpha \tilde{H}} \right).$$

It could be easily verified that the true magnetization is related to the true field $H$ through the Langevin function:

$$M = \frac{B}{\mu_0} - H = M_0 \left( \coth \alpha H - \frac{1}{\alpha H} \right).$$

When written in terms of the true quantities, the true stress (force per unit deformed area) takes the form

$$\sigma = \lambda s = G \left( \lambda^2 - \frac{1}{\lambda^2} \right) + BH - \mu_0 \frac{\lambda}{\alpha} \frac{dM_0}{d\lambda} \ln \frac{\sinh \alpha H}{\alpha H}.$$

It could be recognized that the first term on the right-hand side of Eq. (11) is the stress due to elasticity, and the second term is the axial component of Maxwell stress. The last term in Eq. (11) is the contribution from the strain dependence of permeability.

Utilizing the equations of state (8)–(11), the magneto-mechanical behavior of an MAE, including the magnetostriction, the inverse magnetostriction, and the MR effect, could all be characterized. The central task of characterization is to identify the deformation dependence of magnetization, $M_0(\lambda)$ (and more generally $\alpha(\lambda)$). While the actual functional form should be obtained from experiments, we will illustrate the model through the simple case of small strain, $\varepsilon \ll 1$. The dimensionless
magnetization $\alpha M_0$ can be expanded into a Taylor series of strain $\varepsilon$

$$\alpha M_0(\varepsilon) = 3\chi_0 + a\varepsilon + \frac{b}{2}\varepsilon^2 + \cdots,$$

(12)

where $\chi_0$ is the susceptibility in the undeformed state (the reference state). Considering the manufacturing process of an anisotropic MAE (the one cured under a magnetic field), it could be argued that the coefficient $a \approx 0$. This is because the elastomer is in a liquid state prior to curing, and when a magnetic field is applied the dispersed magnetic particles rearrange into chain-like structures to minimize the then-current total potential energy, which includes the magnetic energy $W_m$ but not the elastic energy $W_e$. The minimization of $W_m$ among all possible configurations at the time of curing suggests an energy minimum at $\varepsilon = 0$ for the cured MAE, $\partial W_m/\partial \varepsilon = 0$ and $\partial^2 W_m/\partial \varepsilon^2 < 0$. Thus, $a \approx 0$ and for the same reason, $b < 0$ holds for materials cured through such a process. On the other hand, for an ideal isotropic sample, in which the distribution of particles is fully random, deformation does not affect particle distribution, and the effective permeability is expected to be a constant: $a = b = 0$. For simplicity, we will omit $a$ in the following calculation. For better characterization of MAEs, especially those cured via different processes, the model can always be extended to include both $a$ and $b$.

Applying the relation $\lambda = 1 + \varepsilon$, the stress expression (11) could be rewritten approximately with the leading order terms of strain $\varepsilon$:

$$\sigma \approx 3G\varepsilon + \mu_0 H^2 + 3\chi_0 \frac{\mu_0 H}{\alpha} \left( \coth \alpha H - \frac{1}{\alpha H} \right) - b\frac{\mu_0}{\alpha^2} \ln \frac{\sinh \alpha H}{\alpha H}.$$

(13)

The MR effect could be obtained directly. The change in Young’s modulus under constant true magnetic field is simply

$$\Delta E = \left. \frac{\partial \Delta \sigma}{\partial \varepsilon} \right|_H = -b\frac{\mu_0}{\alpha^2} \ln \frac{\sinh \alpha H}{\alpha H}.$$

(14)

Equation (14) shows that the MR effect is directly related to the inverse magnetostriction. In the case of small magnetic field, $\alpha H \ll 1$, Eq. (14) could be approximated by keeping the leading terms of $\alpha H$:

$$\Delta E \approx -\frac{b}{6} \mu_0 H^2.$$

(15)

It could be seen from Eq. (15) that the change in stiffness at low magnetic field is proportional to the squared of the field. The sign and magnitude of the MR effect is determined by parameter $b$, and the change in stiffness is always positive for MAEs cured under a magnetic field (i.e., stiffening) since $b < 0$. Moreover, as a second order coupling effect, the stiffness change is independent of the material’s reference susceptibility $\chi_0$.

Although the magnetostriction in the case when the field is uniform (e.g., by submerging the sample to a liquid of the same permeability) could be calculated directly from Eq. (11) or (13), such an idealization is of little practical use for an MAE whose permeability changes with deformation. Instead, we study a case
closer to most experimental setups. Imagine a cylindrical MAE sample placed in a long solenoid, as illustrated in Fig. 2. A constant current is passing through the coil, resulting in a uniform magnetic field prior to the insertion of the sample. The sample is much smaller than the coil and has a large aspect ratio, so that the field inside is close to uniform, while the magnetic field outside the sample is perturbed. From Ampère’s law, we can assume the flux density \( B \) to be continuous across the top and bottom surfaces of the sample, and the magnetic field \( H \) to be continuous across the lateral surface, as illustrated on Fig. 2. In the absence of any mechanical load, the stress of the sample is balanced by the Maxwell stress in the surrounding medium. With a hydrostatic component omitted, the equivalent axial stress takes the form [Bustamante, 2010]

\[
\sigma = \frac{B^2}{2\mu_0} + \frac{\mu_0}{2} H^2. \tag{16}
\]

Equating (13) and (16) and keeping only the leading order terms of strain, we obtain the equilibrium strain:

\[
\varepsilon \approx \frac{9}{2} \chi_0^2 \left( \coth \alpha H - \frac{1}{\alpha H} \right)^2 \frac{G \alpha^2}{\mu_0} - b \ln \sigma \frac{\alpha H}{\alpha H}. \tag{17}
\]

In the low field limit,

\[
\varepsilon \approx \frac{\mu_0 \chi_0^2 H^2}{6G}, \tag{18}
\]

and the magnetostriction is proportional to the squared of the field. The sign of magnetostriction is always positive, i.e., the sample extends along the field direction.

3. Experimental Validation

3.1. Sample preparation and mechanical characterization

MAE samples were prepared by mixing carbonyl iron powder of \( \sim 6 \mu m \) average diameter (ISP S-1641) into a polymer solution of EL P7683/50 A and B (in a
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The mixture was stirred for proper dispersion of the iron particles. Then the suspension was left in a vacuum chamber for 5 min to expel air bubbles. All samples had the same cylindrical shape and the same size with both the diameter and the height being $\frac{1}{2}$ inch. The polymer was cured for 6 h under room temperature. Two groups of MAE samples with different microstructure were made. Isotropic samples were cured in the absence of a magnetic field while anisotropic samples were cured in a uniform magnetic field ($\sim 1250$ Gauss) between a pair of permanent magnets (Amazing magnets, Q250P). For each group of samples, four different concentrations of iron particles were prepared: 20 wt.%, 30 wt.%, 40 wt.% and 50 wt.%. There were altogether eight samples with different microstructures and amount of iron particles.

A series of axial compression tests were conducted using INSTRON 8862. The results were found to be independent of the loading rate within the range of interest. The measured Young’s moduli of the two groups of samples are plotted as a function of the weight fraction of iron particles on Fig. 3. Clearly, with same amount of iron particles, the anisotropic samples have larger modulus. In general, the tensile modulus increases with the particle concentration with some exceptions possibly caused by material inhomogeneity. By assuming the material to be incompressible, its shear modulus $G$ will be taken as one third of the measured Young’s modulus when used in later calculations.

3.2. Permeability and inverse magnetostriction measurement

The relative permeability of the MAE samples was measured by using two magnetometers (GM1 and GM2). The magnetic field was applied through an electromagnet (EMU-75) with the working space between poles much larger than the sample, as shown in Fig. 4. Constant direct current was inputted to the electromagnet to generate a close-to-uniform magnetic field in the working space. GM1 was placed away from the sample. As the samples had relatively low permeability, and were much smaller than the steel cores of the electromagnet, the perturbation to the

![Fig. 3. Measured Young’s moduli of various MAE samples with different iron weight fractions. The error bar represents the nominal accuracy of the gauge.](image-url)
uniform magnetic field was minimal. It was assumed that the magnetic field expe-
rienced by the sample was the same as that by GM1: \( H = B_{\text{GM1}}/\mu_0 \). At the same
time, GM2 was placed in the narrow gap between the sample and one of the electric-
magnet cores to measure the magnetic flux density. The gap was so thin that the
magnetic flux density measured by GM2 could be assumed to be the same as that
through the sample: \( B = B_{\text{GM2}} \). As shown in Fig. 5, by fitting the measured data
to the Langevin function, Eq. (10), we obtained the magnetic property of a sample
expressed in parameters \( M_0 \) and \( \alpha \).

To measure the strain dependence of permeability (i.e., the inverse magnetostric-
tion), we compressed the samples axially by changing the distance between the
two steel electromagnet cores, which constrained the samples. The magnetization
data under different strains, \( \alpha M_0(\varepsilon) \), were then fitted to a polynomial function as
Eq. (12). As only compressive strain could be applied in our experiments, the same
assumption \( a = 0 \) was employed in fitting the data. For each sample, three material
parameters were extracted: \( \alpha \), \( \chi_0 \), and \( b \), and summarized in Table 1. As the measure-
ment involved two consecutive nonlinear fitting steps, the accuracy was hard
to control, especially for the second order coefficient \( b \). As plotted in Fig. 6, the
normalized magnetization \( \alpha M_0 \) only changes slightly within the range of applied
strain. Directly using these data for quantitative prediction may cause inaccuracy,
as will be shown in the following sections. Even so, it is clear that parameter \( b \)
is negative for anisotropic MAEs. Besides, it shows \( b \) is much smaller in isotropic
samples, closer to the ideal case \( a = b = 0 \). All these observations support the key
hypothesis of our model.
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Fig. 5. (a) The magnetization-magnetic-field relation can be approximated by the Langevin function. The slope of the curve at low magnetic field is approximately \( \alpha M_0 / 3 \). (b) Experimental measurement of magnetization of samples with four different iron concentrations. The symbols are the measured data points, and the lines are the fitted curves to the Langevin function, Eq. (10). The solid curves are for isotropic samples, and the dash curves are for anisotropic samples. The anisotropic samples have higher initial slopes and saturate earlier than isotropic samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Isotropic</th>
<th>Anisotropic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha ) (mm/kA)</td>
<td>( \chi_0 )</td>
</tr>
<tr>
<td>20 wt.%</td>
<td>4.41</td>
<td>0.10</td>
</tr>
<tr>
<td>30 wt.%</td>
<td>3.95</td>
<td>0.18</td>
</tr>
<tr>
<td>40 wt.%</td>
<td>3.59</td>
<td>0.27</td>
</tr>
<tr>
<td>50 wt.%</td>
<td>5.57</td>
<td>0.46</td>
</tr>
</tbody>
</table>

3.3. Magnetostriction measurement

The experimental setup for magnetostriction measurement was similar to that for permeability measurement, as shown in Fig. 4. The MAE samples were placed along
Fig. 6. Inverse magnetostriction: Strain dependence of permeability. Susceptibility $\alpha M_0/3$ is plotted as a function of the compressive strain for various samples. The symbols are the measured data points (extracted from fitting the magnetization curve), and the lines are the fitted quadratic functions. The solid lines are for anisotropic samples, and the broken lines are for isotropic samples.

Fig. 7. Measured magnetostriction strain $\Delta L/L$ of (a) isotropic samples and (b) anisotropic samples. The symbols are the measured data from samples of various iron concentrations, and the error bars indicate the range of data in multiple measurements. The continuous curves are the prediction of Eq. (17) from the data of the permeability measurements in Table 1.

the axis of the working space. One end was glued to the surface of a steel core, and the other end was left free. The magnetostriction strain, $\varepsilon = \Delta L/L$, was measured optically through the camera focused on the samples. Figure 7 shows the measured strains of isotropic and anisotropic samples. To compare with the model prediction, the strains are plotted against the dimensionless combination $\mu_0 H^2/G$. The model prediction, Eq. (17), is also plotted on Fig. 6 by applying the material parameters in Table 1. Since all parameters were extracted from an independent and relatively inaccurate experiment of inverse magnetostriction (Sec. 3.2) and no fitting parameter is involved, the consistency between the model predictions and the direct strain measurements is actually quite good. With more carefully designed measurements.
of the inverse magnetostriction or the MR effect (Sec. 3.4), quantitative agreement could be expected.

Because the anisotropic samples have relatively high susceptibility, their magnetostrictive strain under low magnetic field ($\mu_0 H^2/G < 0.25$) increases much faster with the field than that of the isotropic ones. On the other hand, the lower saturation field (higher $\alpha$ value) of anisotropic samples causes their high-field magnetostriction to be much smaller than that of the isotropic ones. A comparison between the experimental results and the model predictions also reveals the significance of the material parameters. Under low field, the second order coefficient $b$ only slightly affects magnetostriction, which is a first order coupling effect. Comparing the results between isotropic and anisotropic samples, one may identify the earlier saturation of magnetostriction strain caused by the nonzero coefficient $b$.

### 3.4. Magneto-rheological effect

Even though the inclusion of an MR-effect measurement naturally completes the characterization of magneto-mechanical coupling and would further validate the model, such a measurement is beyond the experimental capability of our lab. To apply a uniform magnetic field without interference with the force measurement is perhaps the most challenging task. Instead, we will compare our model prediction to some published experimental results.

In Fig. 8, we plot the MR effect predicted by Eq. (14) by using the same material parameters extracted from the inverse magnetostriction measurements in Table 1. For comparison, we also plot the experimental data from Ref. 15 on a very similar material system. As shown by Fig. 8, the model prediction captures the qualitative trend and matches the experiment on the order of magnitude. Just as observed in the experimental data, the anisotropic samples exhibit a more pronounced MR effect than the isotropic ones.

![Fig. 8. Prediction of the stiffness increase of (a) isotropic and (b) anisotropic MAEs as a function of the applied magnetic field. The curves are the prediction of Eq. (14) from the permeability measurement data in Table 1. The open symbols are the experimental results on a similar (but not identical) material system adopted from Ref. 15.](image-url)
experiments, the MR effect of anisotropic MAEs are much more significant than isotropic samples in general. While the results of isotropic samples are closer, the experimental data on the MR effect of anisotropic MAEs are consistently higher than our model prediction, as shown by Fig. 8(b). We believe that the discrepancy is mainly caused by the difference in the curing magnetic field. The anisotropic MAE samples in the current paper, on which the model prediction is based, were cured under a magnetic field of 125 mT, while the samples tested in Ref. 15 were cured under a field of 400 mT. It is known that a higher curing field leads to an MAE with stronger MR effect [Chen et al., 2007]. If the samples are cured under the same conditions, we expect that the model developed in the current paper will predict much more consistent results among all three magneto-mechanical coupling effects. Moreover, by fitting the model to a carefully designed measurement of the MR effect under uniform magnetic fields instead of the less sensitive inverse magnetostriction, more accurate material parameters may be extracted and better agreement could be achieved among all three sets of data.

4. Conclusion

This paper presents a simple constitutive model for MAEs subject to a magnetic field and an axial strain in the same direction. By assuming the magneto-mechanical coupling to be originated from the deformation-dependent magnetization (or permeability), the model has only a few material parameters that could be readily measured, and is capable of predicting the nonlinear response even under high magnetic field. Using the material parameters extracted from the measurement of magnetic permeability under different strains, the model can predict both the magnetostriction and the MR (field-stiffening) effect. The model predictions are validated through our in-house experiments and by comparing to the results in the literature. Good agreement has been achieved among independent experiments of different coupling properties. The success of this model also suggests the physical validity of the basic assumption: The magneto-mechanical coupling behaviors of particle-filled MAEs are dominated by the strain dependency of magnetization energy in the material. The different behaviors of isotropic and anisotropic samples show that the minute inverse magnetostriction (strain-dependent permeability) has strong influence on both MR effect and magnetostriction. Although the model is presented for the simple case of axial magnetic field and deformation, it could be easily extended to more general cases by considering the anisotropy in magnetization and elasticity.

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