1. Suppose that \( X_i \sim N\left(\frac{1}{i}, \left(\frac{1}{i}\right)^2\right) \) (the standard deviation is \( 1/i \)). Prove or disprove that these variables converge in distribution.

2. Suppose that \( \{Y_i\} \) is a sequence of iid random variables with \( \text{E} Y_i = 1 \) and \( \text{Var} Y_i = 1 \). Let \( W_i = \mu_i Y_i \) for each \( i \) and \( S_n = \sum_{i=1}^{n} W_i \).

   a) What is a marginal distribution for \( Y_i \) under which \( (S_n - \text{ES}_n) / \sqrt{\text{Var} S_n} \to^d N(0,1) \) for any (not identically 0) sequence of reals \( \{\mu_i\} \)?

   b) Give a sufficient condition on a decreasing sequence of positive reals \( \{\mu_i\} \) such that for any marginal distribution for \( Y_i \), \( (S_n - \text{ES}_n) / \sqrt{\text{Var} S_n} \to^d N(0,1) \). (Argue that your condition is indeed sufficient.)

3. Consider independent sequences of random variables on the same probability space, \( \{X_n\} \) and \( \{N_n\} \) with the properties that the \( N_n \) are positive integer valued, \( X_n \to^d X \) and \( N_n \to \infty \) a.s.

   Argue carefully that \( X_{N_n} \to^d X \)

   using conditioning and characteristic functions.

4. Suppose that \( \{X_i\} \) is a sequence of random variables taking values in \([a,b]\) for real numbers \( a < b \) and that \( X_i \to^d X \)

   Argue carefully that the moment generating functions for the \( X_i \) converge to that of \( X \).

   That is, argue carefully that for \( t \in \mathbb{R} \)
   \[ \text{E} \exp(tX_i) \to \text{E} \exp(tX) \]
5. Suppose that \( X \sim \text{Exp}(1) \) independent of \( W \sim \text{Ber}\left(\frac{1}{2}\right) \). Find the characteristic function of \( Y = 2WX - X \) and use it to argue that \( Y \) has a distribution symmetric about 0. (The \( \text{Exp}(1) \) characteristic function is \( \phi(t) = (1 - ut)^{-1} \).)

6. Suppose that \( X_1, X_2, \ldots, X_n \) are iid Binomial \((m, p)\) random variables, and neither \( m \geq 1 \) nor \( p \in (0,1) \) is known.

a) Identify a minimal sufficient statistic for the parameter \((m, p)\) and argue carefully that it is indeed minimal sufficient. (This is way harder than I intended. My original "solution" was wrong.)

b) For the case of \( n = 2 \), \( T \) your minimal sufficient statistic from a), and \( g(x_1, x_2) : \{0,1,2,\ldots\}^2 \to \mathbb{R} \), give an explicit formula for 
\[
E\left[ g(X_1, X_2) | \sigma(T) \right]
\]
and demonstrate that your prescription doesn't depend upon the parameter \((m, p)\).

7. Suppose that \( U, V, \) and \( W \) are independent random variables, \( U \sim \text{N}(\theta, 1) \), \( V \sim \text{N}(\theta, 4) \), and \( W \sim \text{Ber}\left(\frac{1}{2}\right) \). Define \( Y = WU + (1 - W)V \) and \( X = (W, Y) \). Prove or disprove that \( X \) is complete and sufficient. (The intention was that the model under consideration is that for \( X \), not that for \((U, V, W)\). But I marked either way.)

8. Use a probabilistic argument to identify \( \lim_{n \to \infty} \left( \cos\left(\frac{t}{\sqrt{n}}\right) \right)^n \). (Argue carefully that your answer is correct.)