Stat 544 Spring 2008
Mini-Project #3

The book *Statistical Quality Assurance for Engineers* by Vardeman and Jobe contains a data set from an ISU Industrial Engineering student project concerned with the operation of a collating machine at a printing facility. There were 2 levels of "Bar Tightness" and 3 levels of "Air Pressure" studied in a 2×3 factorial study. For known "running times" t_{ij} of operation under the 6 different conditions, what was recorded were values

$$y_{ij} = \text{number of collator jams in } t_{ij} \text{ seconds running time with bar tightness } i \text{ and air pressure } j$$

Here are the data

Bar Tightness	Air Pressure	Collator Jams, y	Running Time, t
tight (2)	low (1)	27	295
tight (2)	medium (2)	21	416
tight (2)	high (3)	33	308
loose (1)	low (1)	15	474
loose (1)	medium (2)	6	540
loose (1)	high (3)	11	498

The running times don't include time spent fixing jams, so a plausible model here is that for

$$\lambda_{ij} = \text{mean jams per second with bar tightness } i \text{ and air pressure } j$$

one has

$$Y_{ij} \sim \text{ind Poisson}\left(t_{ij}\lambda_{ij}\right)$$

and we will consider a model that for λ_{11} a (fixed) baseline jam rate, β_2 a (fixed) bar tightness effect for tight bar, α_2 a (fixed) air pressure effect for medium pressure, and α_3 a (fixed) air pressure effect for high pressure, sets

$$\ln\left(\lambda_{ij}\right) = \ln\left(\lambda_{11}\right) + \beta_2 I\left[i=2\right] + \alpha_2 I\left[j=2\right] + \alpha_3 I\left[j=3\right]$$

(I [condition] is 1 or 0 depending upon whether or not "condition" is true.) This is a model with 4 parameters. Do a complete Bayes analysis here including finding credible intervals for all 6 jam rates.

Below are some hypothetical data Vardeman generated to simulate what might happen if an investigator were to rerun this study on five consecutive days, being careful to get all run times equal to 500 seconds.

Bar Tightness	Air Pressure	Day 1	Day 2	Day 3	Day 4	Day 5
tight (2)	low (1)	36	60	38	62	93
tight (2)	medium (2)	17	26	16	34	41
tight (2)	high (3)	34	58	41	52	77
loose (1)	low (1)	7	11	11	10	30
loose (1)	medium (2)	8	8	7	6	6
loose (1)	high (3)	6	15	8	11	24

Suppose that for λ_{110} a baseline jam rate, β_2 a bar tightness effect for tight bar, α_2 an air pressure effect for medium pressure, α_3 an air pressure effect for high pressure, and δ_t a day effect for day t one has (for λ_{ijt} the jam rate for set-up (i, j) on day t)

$$\ln(\lambda_{ijt}) = \ln(\lambda_{110}) + \delta_t + \beta_2 I[i = 2] + \alpha_2 I[j = 2] + \alpha_3 I[j = 3]$$

Consider an analysis that (again) treats the parameters $\lambda_{110}, \beta_2, \alpha_2$, and α_3 as fixed effects, but treats δ_t as iid $\mathrm{N}(0, \sigma^2)$ random effects. (So this model has 5 parameters.) Do another complete Bayes analysis. As part of your analysis, suppose that the machine is going to be run tomorrow and give credible intervals for the jam rates one could expect under each of the 6 possible machine set-ups.

Limit what you type up to turn in to a cover page (**use one**!) plus at most 6 typewritten pages (including whatever figures you want to include). Use at least 11 point fonts, 1.5 line spacing, and 1 inch left and right margins. Also include an Appendix with "commented" `WinBUGS` and/or `R` code that you have used. (This Appendix does not count in the above "6 typewritten pages" limit.)