Suppose that the prior distribution for $\theta$ is $\theta \sim \text{Beta}(\alpha = 4, \beta = 4)$, with pdf
$$f(\theta|4,4) = \frac{\Gamma(4)}{\Gamma(4)^2} \theta^3(1-\theta)^3, \quad 0 \leq \theta \leq 1.$$  

The conditional distribution for $Y$ is $Y|\theta \sim \text{Binomial}(10, \theta)$ with pmf
$$P(Y = y|\theta) = \binom{10}{y} \theta^y(1-\theta)^{10-y}, \quad y = 0, 1, ..., 10$$

Then we have the Joint distribution for $\theta$ and $Y$,
$$f(\theta, Y) = f(Y|\theta)f(\theta) = \binom{10}{y} \frac{7!}{3!3!} \theta^{y+3}(1+\theta)^{13-y}, \quad 0 \leq \theta \leq 1 \text{ and } x = 0, 1, ..., 10$$

After knowing that $Y < 3$, the posterior density for $\theta|Y < 3$ is:
$$f(\theta|Y < 3) = \frac{f(\theta,Y < 3)}{P(Y < 3)} \propto f(\theta,Y = 0) + f(\theta,Y = 1) + f(\theta,Y = 2)$$
$$= \theta^3(1-\theta)^{13} + 10\theta^4(1-\theta)^{12} + 45\theta^5(1-\theta)^{11}$$
$$= \theta^3(1-\theta)^{11}(36\theta^2 + 8\theta + 1), \quad 0 \leq \theta \leq 1$$

**Figure 1:** The curve proportion to the posterior density for $\theta$.

R code for plot:
```r
x = seq(0, 1, 0.01)
y = (x^3) * ((1 - x)^11 * (36 * x^2 + 8 * x + 1))
plot(x, y, type = "l")
```

---

Suppose that $\theta$ has prior distribution $P(\theta = 0.6) = \frac{1}{2}$ and $P(\theta = 0.4) = \frac{1}{2}$. Suppose that the conditional distribution of $X_i|\theta$ is $P(X_i = 1) = \theta = 1 - P(X_i = 0)$ for $i=1$ or 2. So, the joint distribution function for
(X_1, X_2, \theta) is P(x_1, x_2, \theta) = \frac{1}{2} \theta^{x_1+x_2} (1 - \theta)^{2-x_1-x_2} and the marginal distribution for X_1, X_2 (also called the prior predictive distribution) is
\[
P(x_1, x_2) = \frac{1}{2} P(x_1, x_2, 0.6) + \frac{1}{2} P(x_1, x_2, 0.4)
\]
After observing X_1 = X_2 = 0, the posterior distribution of \(\theta | (Y_1, Y_2)\) is
\[
P(\theta | X_1 = 0, X_2 = 0) = \frac{P(X_1 = 0, X_2 = 0 | \theta) P(\theta | X_1 = 0, X_2 = 0)}{P(X_1 = 0, X_2 = 0)} = \frac{(1 - \theta)^2 / 2}{(1 - 0.6)^2 / 2 + (1 - 0.4)^2 / 2} = \frac{(1 - \theta)^2}{0.52}, \text{ for } \theta \in \{0.6, 0.4\}.
\]
Consider a new sequence of trials \(Y_i\) with the same distribution of \(X_i\). Let \(Y = \) the smallest \(i\) such that \(Y_i = 1\). Then \(Y | \theta \sim \text{Geometric}(\theta)\) and \(P(Y = y | \theta) = \theta(1 - \theta)^{y-1}\) for \(y = 1, 2, \ldots\)

The posterior predictive distribution for \(Y | X_1, X_2\) is
\[
P(Y = k | X_1 = 0, X_2 = 0) = \sum_{\theta=0.4,0.6} P(Y = k | \theta) P(\theta | X_1 = 0, X_2 = 0)
\]
\[
= \frac{0.6^2 (0.4)(0.6)^{k-1} + 0.4^2 (0.6)(0.4)^{k-1}}{0.52 (0.6)^{k+1} + 0.52 (0.4)^{k+1}}
\]
So the expectation of \(Y | X_1, X_2\) could be calculated by the following summation
\[
E(Y | X_1 = X_2 = 0) = \sum_{k=0}^{\infty} k \frac{0.6^2 (0.4)^k + 0.4^2 (0.6)^k}{0.52 (0.6)^{k+1} + 0.52 (0.4)^{k+1}}
\]
\[\approx 2.2436\]

GCS&R 2.5

\textbf{Ans:}
Consider \(Y | \theta \sim \text{Bin}(n, \theta)\), with pmf \(P(y | \theta) = \binom{n}{y} \theta^y (1 - \theta)^{n-y}\), \(y = 0, 1, \ldots, n\).

(a)
If the prior distribution of \(\theta\) is Uniform(0, 1), the prior predictive distribution is
\[
P(Y = k) = \int_0^1 P(Y = k | \theta) d\theta = \binom{n}{y} \int_0^1 \theta^k (1 - \theta)^{n-k} d\theta
\]
\[
= \binom{n}{y} \left[ \int_0^1 \theta^k (1 - \theta)^{n-k} d\theta \right] = \binom{n}{y} \frac{1}{n+k+1} \cdot \int_0^1 \theta^k d\theta
\]
\[= \frac{1}{n+1} \]

(b)
If the prior distribution of \(\theta \sim \text{Beta}(\alpha, \beta)\) with pdf \(f(\theta) \propto \theta^{\alpha-1} (1 - \theta)^{\beta-1}\), then the posterior distribution \(\theta | Y \sim \text{Beta}(y+\alpha, n-y+\beta)\) with pdf \(f(\theta | Y) \propto \theta^{y+\alpha-1} (1 - \theta)^{n-y+\beta-1}\) and mean \(\frac{y+\alpha}{n-y+\beta+y+\alpha} = \frac{y+\alpha}{n+\alpha+\beta}\)
If \( \frac{a}{b} \geq \frac{c}{d} \) and \( a, b, c, d > 0 \), then \( \frac{a+c}{b+d} \geq \frac{c}{d} \) if \( a+b = n+2 \Rightarrow ab \leq (\frac{n}{2} + 1)^2 \)

\[
\begin{align*}
\text{(c)} & \quad \text{If the prior distribution of } \theta \sim \text{Uniform}(0,1) \text{ with } Var(\theta) = \frac{1}{12}. \text{ The posterior distribution is } \theta|Y \sim \text{Beta}(y+1, n-y+1) \text{ with pdf } f_{\theta|Y} \propto \theta^y(1-\theta)^{n-y} \\
& \quad \text{then } Var(\theta|Y) = \frac{(y+1)(n-y+1)}{(n+2)^2(n+3)} \text{ if } a+b = n+2 \Rightarrow ab \leq (\frac{n}{2} + 1)^2 \\
& \quad \leq \frac{1}{4(n+3)} \leq \frac{1}{12} \quad \forall n \geq 0
\end{align*}
\]

\[
\begin{align*}
\text{(d)} & \quad \text{If the prior distribution of } \theta \sim \text{Beta}(\alpha, \beta) \text{ with } Var(\theta) = \frac{\alpha \beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \text{ Then the posterior distribution is } \theta|Y \sim \text{Beta}(y+\alpha, n-y+\beta) \text{ with } Var(\theta|Y) = \frac{(y+\alpha)(n-y+\beta)}{(n+\alpha+\beta)^2(n+\alpha+\beta+1)}. \text{ Let } \alpha = 3, \beta = 1, n = 1, \text{ and } y = 0, \text{ then } Var(\theta) = \frac{3^2}{80} < Var(\theta|Y) = \frac{3}{15}. 
\end{align*}
\]

\[
\text{GCS&R 2.8}
\]

\[
\text{Ans:}
\]

Suppose that the prior distribution of \( \theta \sim N(180, 40^2) \) and the conditional distribution of \( Y_1, \ldots, Y_n | \theta \sim iid \text{ N}(\theta, 20^2) \). Average \( \bar{y} = 150 \) is observed. Let \( Y = (Y_1, \ldots, Y_n) \).

\[
\text{(a)}
\]

The posterior distribution of \( \theta|Y \):

\[
\begin{align*}
f(\theta|Y) & = f(\theta, Y)/f(Y) \\
& = f(Y|\theta) \cdot f(\theta)/f(Y) \\
& = \left[ \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}20^2} e^{-\frac{1}{2\cdot20^2}(y_i-\theta)^2} \right] \left[ \frac{1}{\sqrt{2\pi}40^2} e^{-\frac{1}{2\cdot40^2}(\theta-180)^2} \right] / f(Y) \\
& \propto e^{-\frac{1}{2\cdot40^2}(\theta-180)^2} e^{-\frac{1}{2\cdot40^2}(\bar{y}-\theta)^2} / \sqrt{2\pi\sigma^2_{\theta|Y}}
\end{align*}
\]

i.e. \( \theta|Y \sim \text{Normal distribution with mean } \mu_{\theta|Y} = \frac{180+4n\bar{y}}{1+4n} = \frac{180+600n}{1+4n} \text{ and variance } \sigma^2_{\theta|Y} = \frac{40^2}{1+4n}. \)

\[
\text{(b)}
\]

The posterior predictive distribution for a new student’s weight \( \bar{y} \) is

\[
\begin{align*}
f(\bar{y}|Y) & = \int_{-\infty}^{\infty} f(\bar{y}|\theta) f(\theta|Y) d\theta \\
& = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}20^2} e^{-\frac{1}{2\cdot20^2}(\bar{y}-\theta)^2} \frac{1}{\sqrt{2\pi\sigma^2_{\theta|Y}}} e^{-\frac{1}{2\cdot40^2}(\theta-\mu_{\theta|Y})^2} d\theta
\end{align*}
\]

By Textbook page 48, \( \bar{y}|Y \sim N(\mu_{\bar{y}|Y}, \sigma^2_{\bar{y}|Y}) \) where

\[
\begin{align*}
\mu_{\bar{y}|Y} & = E[\bar{y}|Y] = EE[\bar{y}|Y, \theta] = E[\theta|Y] = \frac{180 + 600n}{1 + 4n} = \mu_{\theta|Y}
\end{align*}
\]

---

2008-02-05

Homework 1

STAT544
and 
\[ \sigma^2_{\tilde{Y}|Y} = \text{Var}(\tilde{Y}|Y) = 20^2 + \text{Var}(\theta|Y) = 400 + \frac{40^2}{1 + 4n} = 400 + \sigma^2_{\theta|Y}. \]

(c) 
For \( n = 10 \), 
95% posterior interval for \( \theta \) is 
\[ \frac{180 + 6000}{1 + 40} \pm 1.96 \frac{40}{\sqrt{1 + 40}} = [138, 163]. \]
95% posterior predicted interval for \( \tilde{Y} \) is 
\[ \frac{180 + 6000}{1 + 40} \pm 1.96 \sqrt{20^2 + \frac{40^2}{1 + 40}} = [110, 192]. \]

(d) 
For \( n = 100 \), 
95% posterior interval for \( \theta \) is 
\[ \frac{180 + 60000}{1 + 40} \pm 1.96 \frac{40}{\sqrt{1 + 40}} = [146, 154]. \]
95% posterior predicted interval for \( \tilde{Y} \) is 
\[ \frac{180 + 60000}{1 + 40} \pm 1.96 \sqrt{20^2 + \frac{40^2}{1 + 400}} = [111, 189]. \]

GCS&R 2.12

Ans:

Jeffrey’s principle: define the noninformative prior density \( p(\theta) \propto |J(\theta)|^{1/2} \).
Suppose \( Y|\theta \sim \text{Poisson}(\theta) \) with mean \( \theta \) and variance \( \theta \). The pmf of \( Y|\theta \) is \( P(y|\theta) = \frac{1}{y!} \theta^y e^{-\theta} \) for \( y = 0, 1, 2, \ldots, \) and \( \theta > 0 \) Then the log of pmf \( \log(P(Y|\theta)) = -\log(y!) + y\log(\theta) - \theta \). Take the first and 2nd derivative of log-pmf
\[ \frac{d\log(P(Y|\theta))}{d\theta} = \frac{y}{\theta} - 1, \quad \frac{d^2\log(P(Y|\theta))}{d\theta^2} = -\frac{y}{\theta^2} \]
So \( J(\theta) = -E[-\frac{Y}{\theta^2}|\theta] = \frac{1}{\theta^2} E[Y|\theta] = \frac{\theta}{\theta^2} = \frac{1}{\theta} \). Therefore, \( p(\theta) \propto \frac{1}{\sqrt{\theta}} = \theta^{-1/2} \). This distribution closely matches Gamma(\( \alpha, \beta \)) with \( \alpha = 1/2 \) and \( \beta \approx 0 \).

GCS&R 2.21

Ans:

(a) 
Suppose the conditional distribution is \( Y_1, \ldots, Y_n|\theta \overset{iid}{\sim} \text{Exp}(\theta) \), \( f_{Y_i|\theta} = \theta e^{-y_i \theta} \) for \( y_i > 0 \) and the prior distribution is \( \theta \sim \text{Gamma}(\alpha, \beta) \), \( f_\theta = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta} \) for \( \theta > 0 \). Let \( Y = (Y_1, \ldots, Y_n) \).
\[ f_{\theta|Y} \propto \prod_{i=1}^{n} f_{Y_i|\theta} \cdot f_\theta \]
\[ \propto \theta^{\alpha+n-1} e^{-\left(\sum_{i=1}^{n} y_i + \beta \right) \theta} \]
This is Gamma(\( \alpha + n, \beta + \sum_{i=1}^{n} y_i \)). So the gamma prior distribution is conjugate for inference about \( \theta \) given an iid sample of \( y \) values.

(b) 
For the mean \( \phi = \frac{1}{\theta} \),
\[ f_\phi(\phi) = f_\theta(\phi) \left| \frac{d\theta}{d\phi} \right| = \frac{\beta^\alpha}{\Gamma(\alpha)} \left( \frac{1}{\phi} \right)^{\alpha-1} e^{-\beta/\phi} = \frac{\beta^\alpha}{\Gamma(\alpha)} \phi^{-(\alpha+1)} e^{-\beta/\phi}. \]

This is the distribution function for \( IG(\alpha, \beta). \)

(c)

Suppose the length of life of a light bulb \( Y|\theta \sim \text{Exp}(\theta) \) and the prior distribution \( \theta \sim \text{Gamma} \) with \( \frac{\text{SD}}{\text{mean}} = 0.5 = \frac{\sqrt{\alpha/\beta}}{\alpha/\beta} = \frac{1}{\sqrt{\alpha}} \Rightarrow \alpha = 4. \) A random sample \( Y_1, \ldots, Y_n \) drawn from \( \text{Exp}(\theta) \). The posterior distribution for \( \theta|Y \) is:

\[ f(\theta|Y) \propto \prod_{i=1}^{n} f(Y_i|\theta) f(\theta) \propto \theta^n e^{-\theta \sum y_i} \cdot \theta^{\alpha-1} e^{-\beta \theta} = \theta^{n+\alpha-1} e^{-\theta (\beta + \sum y_i)} \]

i.e. \( \theta|Y \sim \text{Gamma}(n + \alpha, \beta + \sum_{i=1}^{n} y_i) \). So the \( \text{CV} = \frac{1}{\sqrt{n+\alpha}} = 0.1 \Rightarrow n = 96. \)

(d)

Since \( \phi \sim IG(\alpha, \beta) \), the coefficient of variation refers to \( \phi \) is \( (\alpha - 2)^{-1/2} \equiv c \Rightarrow \alpha = 2 + 1/c^2 \). Further, \( \phi|Y \sim IG(n + \alpha, \beta + \sum_{i=1}^{n} y_i) \), \( \text{CV} \) of \( \phi|Y \) is \( (n + \alpha - 2)^{-1/2} \equiv d \Rightarrow n = 1/d^2 - 1/c^2 \).

If \( c = 0.5, d = 0.1 \) as settings in part (c), then \( n = 96. \)

\begin{center}
\textbf{GCS&R 2.22}
\end{center}

\textit{Ans:}

(a)

Suppose the conditional distribution \( Y|\theta \sim \text{Exp}(\theta), P(Y \geq y) = -e^{-\theta x}|_y = e^{-y\theta}, \theta > 0. \) and the prior distribution \( \theta \sim \text{Gamma}(\alpha, \beta), f_\theta = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta \theta} \)

\[ f(\theta|Y \geq 100) = \frac{f(\theta, Y \geq 100)}{P(Y \geq 100)} = \frac{P(Y \geq 100|\theta)f(\theta)}{P(Y \geq 100)}. \]

\[ \theta|Y \geq 100 \sim \text{Gamma}(\alpha + 1, \beta + 100) \) with mean = \( \frac{\alpha}{\beta + 100} \) and \( \text{var} = \frac{\alpha}{(\beta + 100)^2} \)

(b)

Suppose \( Y = 100 \) is observed. \( f_{\theta|Y=100} \propto \theta e^{-100\theta} \theta^{\alpha-1} e^{-\beta \theta} \)

\( \theta|Y = 100 \sim \text{Gamma}(\alpha + 1, \beta + 100) \) with mean = \( \frac{\alpha+1}{\beta + 100} \) and \( \text{var} = \frac{\alpha+1}{(\beta + 100)^2} \)

(c)

\[ \text{Var}(\theta) = E[\text{Var}[\theta|Y]] + \text{Var}[E[\theta|Y]] \Rightarrow \text{Var}(\theta|Y \geq 100) \geq E[\text{Var}[\theta|Y = y]]. \] The right hand side is an average. \( \text{Var}[\theta|Y = 100] \) may or may not smaller than the left hand side. So the equition is not violated.
Question 2

Ans:
(a)
Suppose $Y_1, Y_2 \sim \text{Poisson}(\lambda)$. $f(Y = y|\lambda) = \frac{1}{y!}\lambda^y e^{-\lambda}$, $\lambda > 0$, $y = 0, 1, 2, 3, \ldots$
and the prior distribution $\lambda \sim G(\alpha, \beta)$, $f(\lambda|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta \lambda}$, $\lambda > 0$.
The posterior distribution for $\lambda|Y$ is:

$$f(\lambda|Y = y_1) \propto f(Y = y_1|\lambda)f(\lambda|\alpha, \beta) = \frac{1}{y_1!} \lambda^{y_1} e^{-\lambda} \times \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta \lambda}$$

$$\propto \lambda^{y_1+\alpha-1} e^{-(\beta+1) \lambda}$$

$$\sim G(y_1 + \alpha, \beta + 1).$$

The prior predictive probability for $Y_2$ (marginal of $Y_2$) is:

$$f(Y_2) = \int_0^\infty f(Y_2|\lambda)f(\lambda|\alpha, \beta)d\lambda$$

$$= \int_0^\infty \frac{1}{y_2!} \lambda^{y_2} e^{-\lambda} \times \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta \lambda} d\lambda$$

$$= \frac{y_2!}{\beta^\alpha} \Gamma(\alpha) \int_0^\infty \lambda^{y_2+\alpha-1} e^{-(\beta+1) \lambda} d\lambda$$

$$= \frac{(y_2 + \alpha - 1)\beta \alpha y_2}{\beta + 1} \left( \frac{1}{\beta + 1} \right)^{y_2}$$

$$\sim \text{Neg-bin}(\alpha, \beta).$$

Posterior predictive distribution for $Y_2|Y_1 = y_1$:

$$f(Y_2|Y_1 = y_1) = \int_0^\infty f(Y_2|\lambda, Y_1)f(\lambda|Y_1)d\lambda$$

$$= \int_0^\infty f(Y_2|\lambda)f(\lambda|Y_1)d\lambda \quad (Y_1, Y_2 \text{ are independent})$$

$$= \int_0^\infty \frac{1}{y_2!} \lambda^{y_2} e^{-\lambda} \times \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{y_1+\alpha-1} e^{-(\beta+1) \lambda} d\lambda$$

$$= \frac{1}{\beta^\alpha} \Gamma(\alpha) \int_0^\infty \lambda^{y_2+y_1+\alpha-1} e^{-(\beta+2) \lambda} d\lambda$$

$$= \frac{y_2!}{\beta^\alpha} \Gamma(\alpha) \int_0^\infty \lambda^{y_2+y_1+\alpha-1} e^{-(\beta+2) \lambda} d\lambda$$

$$= \frac{(y_2 + y_1 + \alpha - 1)\beta \alpha y_2}{(\beta + 2)^{y_2+1}} \left( \frac{1}{\beta + 2} \right)^{y_2}$$

$$\sim \text{Neg-bin}(y_1 + \alpha, \beta + 1).$$

Refer to the textbook (GCS&R) page 576, Neg-bin(alpha,beta) is correct, and there is no need to keep alpha as a positive integer.

If we use the form of negative binomial w.r.t. Statistical Inference (Casella & Berger), then it is Neg-bin(alpha, beta/(beta+1)). It is still possible to extend alpha to the case of a positive real number.

By Statistical Inference (Casella & Berger), it is Neg-bin(y1+alpha, (beta+1)/(beta+2)). Both alpha and beta could be positive real numbers.
**Figure 2:** The solid line and dash line represent the prior and posterior density function for \( \lambda \) when \((\alpha, \beta) = (1, 1)\). The dot line and dot-dash line represent densities for \( \lambda \) when \((\alpha, \beta) = (10, 10)\), respectively. Both posterior distributions are lightly pulled to left when observed \( Y = 0 \).

**Figure 3:** The two figures in the first row show the prior predictive distribution of \( f(Y_2) \), and 2nd row shows the posterior predictive distribution \( f(Y_2|Y_1 = 0) \) for \( Y_2 = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \) with \((\alpha, \beta) = (1, 1)\) and \((\alpha, \beta) = (10, 10)\), respectively.
Now consider another prior distribution for $\lambda$: $\lambda \sim U(0, 10)$, $f(\lambda) = \frac{1}{10}$, $\lambda \in [0, 10]$.

The posterior distribution for $\lambda | Y_1 = y_1$:

$$f(\lambda | Y_1 = y_1) \propto f(Y_1 = y_1 | \lambda) f(\lambda) = \frac{1}{y_1!} \lambda^{y_1} e^{-\lambda} \times \frac{1}{10} \propto \lambda^{y_1} e^{-\lambda}, 0 \leq \lambda \leq 10.$$ 

After observing $Y_1 = 0$, the posterior distribution of $\lambda | Y_1 = 0$ is $e^{-\lambda} / (1 - e^{-10})$.

**Figure 4:** The solid line shows the prior distribution and the dash line shows the posterior distribution for $\lambda$.

R code for plot:

```r
x = seq(0, 10, 0.01)
plot(x, exp(-x)/(1-exp(-10)), type = "l")
plot(x, rep(1/10, length(x)))
```

(c) Use WinBUGS to show the approximated posterior distribution.

i. The densities $f(Y_2 | Y_1 = 3)$ and $f(\lambda | Y_1 = 3)$, using the gamma prior for $\lambda$ and $(\alpha, \beta) = (1, 1)$.
ii. The densities \( f(Y_2|Y_1 = 3) \) and \( f(\lambda|Y_1 = 3) \), using the gamma prior for \( \lambda \) and \((\alpha, \beta) = (10, 10)\). 

iii. The density for the posterior \( f(Y_2|Y_1 = 3) \) and \( f(\lambda|Y_1 = 3) \), using the prior \( U(0, 10) \) for \( \lambda \). 

(d) 

i. The density for the posterior \( f(Y_2|Y_1 = 7) \) and \( f(\lambda|Y_1 = 7) \), using the gamma prior for \( \lambda \) and \((\alpha, \beta) = (1, 1)\).
ii. The density for the posterior $f(Y_2|Y_1 = 7)$ and $f(\lambda|Y_1 = 7)$, using the gamma prior for $\lambda$ and $(\alpha, \beta) = (10, 10)$.

iii. The density for the posterior $f(Y_2|Y_1 = 7)$ and $f(\lambda|Y_1 = 7)$, using the prior $U(0, 10)$ for $\lambda$. 

---

Node statistics

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<th>median</th>
<th>97.5%</th>
<th>start</th>
<th>sample</th>
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</thead>
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<td>1.411</td>
<td>0.004235</td>
<td>3.641</td>
<td>7.200</td>
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<td>3.641</td>
<td>7.200</td>
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Kernel density

Y2 sample: 100000

Lambda sample: 100000

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Kernel density

Y2 sample: 100000

Lambda sample: 100000

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