GCS&R 2.1

<u>*Ans*</u>:

Suppose that the prior distribution for θ is $\theta \sim \text{Beta}(\alpha = 4, \beta = 4)$, with pdf

$$f(\theta|4,4) = \frac{\Gamma(8)}{\Gamma(4)\Gamma(4)}\theta^3(1-\theta)^3,\ 0 \le \theta \le 1.$$

The conditional distribution for Y is $Y|\theta \sim \text{Binomial}(10, \theta)$ with pmf

$$P(Y = y|\theta) = \binom{10}{y}\theta^y(1-\theta)^{10-y},\ y = 0, 1, ..., 10$$

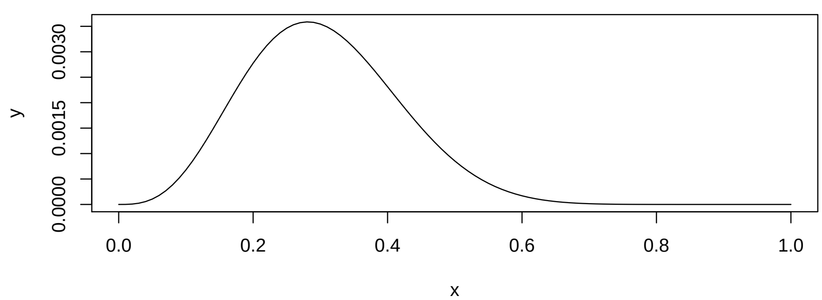
Then we have the Joint distribution for θ and Y,

$$f(\theta, Y) = P(Y|\theta)f(\theta) = \binom{10}{y}\frac{7!}{3!3!}\theta^{3+y}(1+\theta)^{13-y}\ \text{, for } 0 \le \theta \le 1 \text{ and } x = 0, 1, ..., 10$$

After knowning that $Y < 3$, the posterior density for $\theta|(Y < 3)$ is :

$$\begin{aligned} f(\theta|Y<3) &= \frac{f(\theta, Y<3)}{P(Y<3)} \propto f(\theta, Y = 0) + f(\theta, Y = 1) + f(\theta, Y = 2) \\ &= \theta^3(1-\theta)^{13} + 10\theta^4(1-\theta)^{12} + 45\theta^5(1-\theta)^{11} \\ &= \theta^3(1-\theta)^{11}(36\theta^2 + 8\theta + 1), 0 \le \theta \le 1 \end{aligned}$$

Figure 1: The curve proportion to the posterior density for θ.



R code for plot:

```
x = seq(0, 1, 0.01)
y = (x^3) * ((1 - x)^11 * (36 * x^2 + 8 * x + 1))
plot(x, y, type = "l")
```

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GCS&R 2.2

<u>*Ans*</u>:

Suppose that θ has prior distribution $P(\theta = 0.6) = \frac{1}{2}$ and $P(\theta = 0.4) = \frac{1}{2}$. Suppose that the conditional distribution of $X_i|\theta$ is $P(X_i = 1) = \theta = 1 - P(X_i = 0)$ for i=1 or 2. So, the joint distribution function for

(X_1, X_2, θ) is $P(x_1, x_2, \theta) = \frac{1}{2}\theta^{x_1+x_2}(1-\theta)^{2-x_1-x_2}$ and the marginal distribution for X_1, X_2 (also called the prior predictive distribution) is

$$P(x_1, x_2) = \frac{1}{2}P(x_1, x_2, 0.6) + \frac{1}{2}P(x_1, x_2, 0.4)$$

After observing $X_1 = X_2 = 0$, the posterior distribution of $\theta|(Y_{1,Y}2)$ is

$$P(\theta|X_1 = 0, X_2 = 0) = \frac{P(X_1 = 0, X_2 = 0, \theta)}{P(X_1 = 0, X_2 = 0)} = \frac{(1-\theta)^2/2}{(1-0.6)^2/2 + (1-0.4)^2/2} = \frac{(1-\theta)^2}{0.52}, \text{ for } \theta \in \{0.6, 0.4\}.$$

Consider a new sequence of trials Y_i with the same distribution of X_i . Let $Y =$ the smallest i such that $Y_i = 1$. Then $Y|\theta \sim \text{Geometric}(\theta)$ and $P(Y = y|\theta) = \theta(1-\theta)^{y-1}$ for $y = 1, 2, \ldots$

The posterior predictive distribution for $Y|X_1, X_2$ is

$$\begin{aligned}
P(Y = k|X_1 = 0, X_2 = 0) &= \textstyle\sum_{\theta=0.4,0.6} P(Y = k|\theta)P(\theta|X_1 = 0, X_2 = 0) \\
&= \tfrac{0.6^2}{0.52}(0.4)(0.6)^{k-1} + \tfrac{0.42}{0.52}(0.6)(0.4)^{k-1} \\
&= \tfrac{0.4}{0.52}(0.6)^{k+1} + \tfrac{0.6}{0.52}(0.4)^{k+1}
\end{aligned}$$

So the expectation of $Y|X_1, X_2$ could be calculated by the following summation

$$\begin{aligned}
E(Y|X_1 = X_2 = 0) &= \textstyle\sum_{k=1}^{\infty} kP(Y = k|X_1 = X_2 = 0) \\
&= \tfrac{1}{0.52}\textstyle\sum_{k=1}^{\infty}[0.6 \cdot 0.4^{k+1} \cdot k + 0.4 \cdot 0.6^{k+1} \cdot k] \\
&= \tfrac{1}{0.52}[0.4^2 \cdot \tfrac{1}{0.6} + 0.6^2 \cdot \tfrac{1}{0.4}] \\
&\approx 2.2436
\end{aligned}$$

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GCS&R 2.5

Ans:

Consier $Y|\theta \sim \text{Bin}(n, \theta)$, with pmf $P(y|\theta) = \binom{n}{y}\theta^y(1-\theta)^{n-y}$, $y = 0, 1, ..., n$.

(a)

If the prior distribution of θ is Uniform$(0, 1)$, the prior predictive distributoin is

$$\begin{aligned}
P(Y = k) &= \textstyle\int_0^1 P(Y = k|\theta) \cdot 1d\theta \\
&= \tbinom{n}{y}\textstyle\int_0^1 \theta^k(1-\theta)^{n-k}d\theta \\
&= \tbinom{n}{y}[\tfrac{1}{k+1}\theta^{k+1}(1-\theta)^{n-k}|_0^1 + \textstyle\int_0^1 \tfrac{n-k}{k+1}\theta^{k+1}(1-\theta)^{n-k-1}d\theta] \\
&= \tbinom{n}{y}\tfrac{(n-k)!}{n!/k!} \cdot \textstyle\int_0^1 \theta^n d\theta \\
&= \tfrac{1}{n+1}
\end{aligned}$$

(b)

If the prior distribution of $\theta \sim \text{Beta}(\alpha, \beta)$ with pdf $f(\theta) \propto \theta^{\alpha-1}(1-\theta)^{\beta-1}$, then the posterior distribution $\theta|Y \sim \text{Beta}(y+\alpha, n-y+\beta)$ with pdf $f(\theta|Y) \propto \theta^{y+\alpha-1}(1-\theta)^{n-y+\beta-1}$ and mean $\frac{y+\alpha}{(n-y+\beta)+(y+\alpha)} = \frac{y+\alpha}{n+\alpha+\beta}$

If $\frac{a}{b} \geq \frac{c}{d}$ and $a, b, c, d > 0$, then $\frac{a+c}{b+d} - \frac{c}{d} = \frac{ad-cb}{d(b+d)} \geq 0$ and $\frac{a+c}{b+d} - \frac{a}{b} = \frac{cb-ad}{b(b+d)} \geq 0$. So, let $a = y$, $b = n$, $c = \alpha$ and $d = \alpha + \beta$, then $\min(\frac{y}{n}, \frac{\alpha}{\alpha+\beta}) \leq \frac{y+\alpha}{n+\alpha+\beta} \leq \max(\frac{y}{n}, \frac{\alpha}{\alpha+\beta})$

(c)

If the prior distribution of $\theta \sim$ Uniform$(0,1)$ with $Var(\theta) = \frac{1}{12}$. The posterior distribution is $\theta|Y \sim$ Beta$(y+1, n-y+1)$ with pdf $f_{\theta|Y} \propto \theta^y(1-\theta)^{n-y}$

$$\begin{aligned} Var(\theta|Y) &= \frac{(y+1)(n-y+1)}{(n+2)^2(n+3)} \qquad \text{if } a+b=n+2 \Rightarrow ab \leq (\tfrac{n}{2}+1)^2 \\ &\leq \frac{1}{4(n+3)} \\ &\leq \frac{1}{12} \qquad \forall n \geq 0 \end{aligned}$$

(d)

If the prior distribution of $\theta \sim$ Beta(α, β) with $Var(\theta) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$. Then the posterior distribution is $\theta|Y \sim$ Beta$(y+\alpha, n-y+\beta)$ with $Var(\theta|Y) = \frac{(y+\alpha)(n-y+\beta)}{(n+\alpha+\beta)^2(n+\alpha+\beta+1)}$. Let $\alpha = 3$, $\beta = 1$, $n = 1$, and $y = 0$, then $Var(\theta) = \frac{3}{80} < Var(\theta|Y) = \frac{3}{75}$. ■

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GCS&R 2.8

Ans:

Suppose that the prior distribution of $\theta \sim$ N$(180, 40^2)$ and the conditional distribution of $Y_1, \ldots, Y_n|\theta \stackrel{iid}{\sim}$ N$(\theta, 20^2)$. Average $\bar{y} = 150$ is observed. Let $Y = (Y_1, \ldots, Y_n)$.

(a)

The posterior distribution of $\theta|Y$:

$$\begin{aligned} f(\theta|Y) &= f(\theta, Y)/f(Y) \\ &= f(Y|\theta) \cdot f(\theta)/f(Y) \\ &= \left[\prod_{i=1}^n \frac{1}{\sqrt{2\pi}20} e^{-\frac{1}{2\cdot 20^2}(y_i-\theta)^2}\right]\left[\frac{1}{\sqrt{2\pi}40} e^{-\frac{1}{2\cdot 40^2}(\theta-180)^2}\right]/f(Y) \\ &\propto e^{-\frac{1+4n}{2\cdot 40^2}(\theta - \frac{180+4n\bar{y}}{1+4n})^2} \end{aligned}$$

i.e. $\theta|Y \sim$ Normal distribution with mean $\mu_{\theta|Y} = \frac{180+4n\bar{y}}{1+4n} = \frac{180+600n}{1+4n}$ and variance $\sigma^2_{\theta|Y} = \frac{40^2}{1+4n}$.

(b)

The posterior predictive distribution for a new student's weight $\tilde{y}$ is

$$\begin{aligned} f(\tilde{Y}|Y) &= \int_{-\infty}^{\infty} f(\tilde{Y}|\theta) f(\theta|Y) d\theta \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}20} e^{-\frac{1}{2\cdot 20^2}(\tilde{y}-\theta)^2} \frac{1}{\sqrt{2\pi}\sigma_{\theta|Y}} e^{-\frac{1}{2\cdot\sigma^2_{\theta|Y}}(\theta-\mu_{\theta|Y})^2} d\theta \end{aligned}$$

By Textbook page 48, $\tilde{Y}|Y \sim$ N$(\mu_{\tilde{Y}|Y}, \sigma^2_{\tilde{Y}|Y})$ where

$$\mu_{\tilde{Y}|Y} = E[\tilde{Y}|Y] = EE[\tilde{Y}|Y, \theta] = E[\theta|Y] = \frac{180+600n}{1+4n} = \mu_{\theta|Y}$$

and

$$\sigma^2_{\tilde{Y}|Y} = Var(\tilde{Y}|Y) = 20^2 + Var(\theta|Y) = 400 + \frac{40^2}{1+4n} = 400 + \sigma^2_{\theta|Y}.$$

(c)

For $n = 10$,

95% posterial interval for θ is $\frac{180+6000}{1+40} \pm 1.96\frac{40}{\sqrt{1+40}} = [138, 163]$.

95% posterial predicted interval for $\tilde{Y}$ is $\frac{180+6000}{1+40} \pm 1.96\sqrt{20^2 + \frac{40^2}{1+40}} = [110, 192]$.

(d)

For $n = 100$,

95% posterial interval for θ is $\frac{180+60000}{1+40} \pm 1.96\frac{40}{\sqrt{1+40}} = [146, 154]$.

95% posterial predicted interval for $\tilde{Y}$ is $\frac{180+60000}{1+40} \pm 1.96\sqrt{20^2 + \frac{40^2}{1+400}} = [111, 189]$. ∎

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GCS&R 2.12

Ans:

Jeffrey's principle: define the noninformative prior density $p(\theta) \propto [J(\theta)]^{1/2}$.

Suppose $Y|\theta \sim$ Poisson(θ) with mean θ and variance θ. The pmf of $Y|\theta$ is $P(y|\theta) = \frac{1}{y!}\theta^y e^{-\theta}$ for $y = 0, 1, 2, \ldots,$ and $\theta > 0$ Then the log of pmf $log(P(Y|\theta)) = -log(y!) + ylog(\theta) - \theta$. Take the first and 2nd derivative of log-pmf

$$\frac{dlog(P(Y|\theta))}{d\theta} = \frac{y}{\theta} - 1 \ , \ \frac{d^2log(P(Y|\theta))}{d\theta^2} = -\frac{y}{\theta^2}$$

So $J(\theta) = -E[-\frac{Y}{\theta^2}|\theta] = \frac{1}{\theta^2}E[Y|\theta] = \frac{\theta}{\theta^2} = \frac{1}{\theta}$. Therefore, $p(\theta) \propto \frac{1}{\sqrt{\theta}} = \theta^{-1/2}$. This distribution closely matchs Gamma(α, β) with $\alpha = 1/2$ and $\beta \approx 0$. ∎

. . . .

GCS&R 2.21

Ans:

(a)

Suppose the conditional distribution is $Y_1, \ldots, Y_n|\theta \overset{iid}{\sim}$ Exp(θ), $f_{Y_i|\theta} = \theta e^{-y_i\theta}$ for $y_i > 0$ and the prior distribution is $\theta \sim$ Gamma(α, β), $f_\theta = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}$ for $\theta > 0$. Let $Y = (Y_1, \ldots, Y_n)$.

$$\begin{aligned} f_{\theta|Y} &\propto \textstyle\prod_{i=1}^n f_{Y_i|\theta} \cdot f_\theta \\ &\propto \theta^{\alpha+n-1}e^{-(\sum_i y_i+\beta)\theta} \end{aligned}$$

This is Gamma($\alpha + n, \beta + \sum_{i=1}^n y_i$). So the gamma prior distribution is conjugate for inference about θ given an iid sample of y values.

(b)

For the mean $\phi = \frac{1}{\theta}$,

$$f_\phi(\phi) = f_\theta(\phi)|\frac{d\theta}{d\phi}| = \frac{\beta^\alpha}{\Gamma(\alpha)}(\frac{1}{\phi})^{\alpha-1}e^{-\beta/\phi}| - \frac{1}{\phi^2}| = \frac{\beta^\alpha}{\Gamma(\alpha)}\phi^{-(\alpha+1)}e^{-\beta/\phi}.$$

This is the distribution function for $IG(\alpha, \beta)$.

(c)

Suppose the length of life of a light bulb $Y|\theta \sim \text{Exp}(\theta)$ and the prior distributio $\theta \sim$ Gamma with $\frac{\text{SD}}{\text{mean}} = 0.5 = \frac{\sqrt{\alpha}/\beta}{\alpha/\beta} = \frac{1}{\sqrt{\alpha}} \Rightarrow \alpha = 4$. A random sample $Y_1, \ldots Y_n$ drawn from $\text{Exp}(\theta)$. The posterior distribution for $\theta|Y$ is:

$$f(\theta|Y) \propto \prod_{i=1}^n f(Y_i|\theta)f(\theta) \propto \theta^n e^{-\theta\sum y_i} \cdot \theta^{\alpha-1}e^{-\beta\theta} = \theta^{n+\alpha-1}e^{-\theta(\beta+\sum y_i)}$$

i.e. $\theta|Y \sim \text{Gamma}(n+\alpha, \beta + \sum_{i=1}^n y_i)$. So the CV $= \frac{1}{\sqrt{n+\alpha}} = 0.1 \Rightarrow n = 96$.

(d)

Since $\phi \sim \text{IG}(\alpha, \beta)$, the coefficient of variation refers to ϕ is $(\alpha - 2)^{-1/2} \equiv c \Rightarrow \alpha = 2 + 1/c^2$.
Further, $\phi|Y \sim \text{IG}(n+\alpha, \beta + \sum_{i=1}^n y_i)$, CV of $\phi|Y = (n+\alpha-2)^{-1/2} \equiv d \Rightarrow n = 1/d^2 - 1/c^2$.
If $c = 0.5$, $d = 0.1$ as settings in part (c), then $n = 96$. ■

. . . .

GCS&R 2.22

Ans:

(a)

Suppose the conditional distribution $Y|\theta \sim \text{Exp}(\theta)$, $P(Y \geq y) = -e^{-\theta x}|_y^\infty = e^{-y\theta}$, $\theta > 0$. and the prior distribution $\theta \sim \text{Gamma}(\alpha, \beta)$, $f_\theta = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}$

$$\begin{aligned} f(\theta|Y \geq 100) &= f(\theta, Y \geq 100)/P(Y \geq 100) \\ &= P(Y \geq 100|\theta)f(\theta)/P(Y \geq 100) \; . \\ &\propto \theta^{\alpha-1}e^{-(\beta+100)\theta} \end{aligned}$$

$\theta|Y \geq 100 \sim \text{Gamma}(\alpha, \beta + 100)$ with mean $= \frac{\alpha}{\beta+100}$ and var $= \frac{\alpha}{(\beta+100)^2}$

(b)

Suppose $Y = 100$ is observed. $f_{\theta|Y=100} \propto \theta e^{-100\theta}\theta^{\alpha-1}e^{-\beta\theta}$
$\theta|Y = 100 \sim \text{Gamma}(\alpha+1, \beta+100)$ with mean $= \frac{\alpha+1}{\beta+100}$ and var $= \frac{\alpha+1}{(\beta+100)^2}$

(c)

$Var(\theta) = E[Var[\theta|Y]] + Var[E[\theta|Y]] \Rightarrow Var(\theta|Y \geq 100) \geq E[Var[\theta|Y = y]]$. The right hand side is an average. $Var[\theta|Y = 100]$ may or may not smaller than the left hand side. So the equition is not violated.

■

Question 2

<u>*Ans*</u>:

(a)

Suppose $Y_1, Y_2 \overset{\text{iid}}{\sim} \text{Poisson}(\lambda)$, $f(Y = y|\lambda) = \frac{1}{y!}\lambda^y e^{-\lambda}$, $\lambda > 0$, $y = 0, 1, 2, 3, \ldots$.
and the prior distribution $\lambda \sim \text{G}(\alpha, \beta)$, $f(\lambda|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}$, $\lambda > 0$.
The posterior distribution for $\lambda|Y$ is:

$$\begin{aligned}
f(\lambda|Y_1 = y_1) &\propto f(Y_1 = y_1|\lambda)f(\lambda|\alpha, \beta) \\
&= \frac{1}{y_1!}\lambda^{y_1}e^{-\lambda} \times \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda} \\
&\propto \lambda^{y_1+\alpha-1}e^{-(\beta+1)\lambda} \\
&\sim \text{G}(y_1 + \alpha, \beta + 1).
\end{aligned}$$

The prior predictive probability for Y_2 (marginal of Y_2) is:

$$\begin{aligned}
f(Y_2) &= \int_0^\infty f(Y_2|\lambda)f(\lambda|\alpha, \beta)d\lambda \\
&= \int_0^\infty \frac{1}{y_2!}\lambda^{y_2}e^{-\lambda} \times \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}d\lambda \\
&= \frac{1}{y_2!}\frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty \lambda^{y_2+\alpha-1}e^{-(\beta+1)\lambda}d\lambda \\
&= \frac{1}{y_2!}\frac{\beta^\alpha}{\Gamma(\alpha)}\frac{\Gamma(y_2+\alpha)}{(\beta+1)^{y_2+\alpha}} \\
&= \binom{y_2+\alpha-1}{y_2}\left(\frac{\beta}{\beta+1}\right)^\alpha\left(\frac{1}{\beta}\right)^{y_2} \\
&\sim \text{Neg-bin}(\alpha, \beta).
\end{aligned}$$

Posterios predictive distribution for $Y_2|Y_1 = y_1$:

$$\begin{aligned}
f(Y_2|Y_1 = y_1) &= \int_0^\infty f(Y_2|\lambda, Y_1)f(\lambda|Y_1)d\lambda \\
&= \int_0^\infty f(Y_2|\lambda)f(\lambda|Y_1)d\lambda \qquad (Y_1,\ Y_2 \text{ are independent}) \\
&= \int_0^\infty \frac{1}{y_2!}\lambda^{y_2}e^{-\lambda} \times \frac{(\beta+1)^{y_1+\alpha}}{\Gamma(y_1+\alpha)}\lambda^{y_1+\alpha-1}e^{-(\beta+1)\lambda}d\lambda \\
&= \frac{1}{y_2!}\frac{(\beta+1)^{y_1+\alpha}}{\Gamma(y_1+\alpha)}\int_0^\infty \lambda^{y_2+y1+\alpha-1}e^{-(\beta+2)\lambda}d\lambda \\
&= \frac{1}{y_2!}\frac{(\beta+1)^{y_1+\alpha}}{\Gamma(y_1+\alpha)}\frac{\Gamma(y_2+y_1+\alpha)}{(\beta+2)^{y_2+y_1+\alpha}} \\
&= \binom{y_2+y_1+\alpha-1}{y_2}\left(\frac{\beta+1}{\beta+2}\right)^{y_1+\alpha}\left(\frac{1}{\beta+1}\right)^{y_2} \\
&\sim \text{Neg-bin}(y_1+\alpha, \beta+1).
\end{aligned}$$

Figure 2: The solid line and dash line represent the prior and posterior density function for λ when $(\alpha, \beta) = (1, 1)$. The dot line and dot-dash line represent densities for λ when $(\alpha, \beta) = (10, 10)$, respectively. Both posterior distributions are lightly pulled to left when observed $Y = 0$.

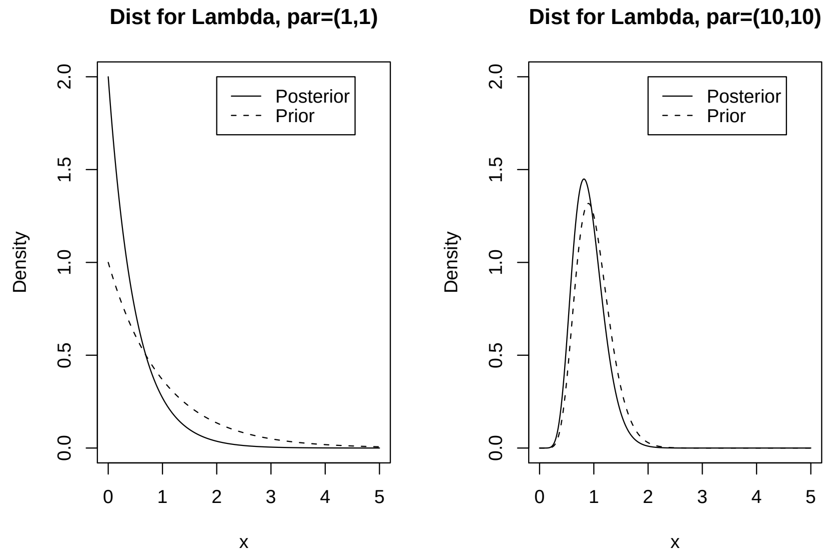
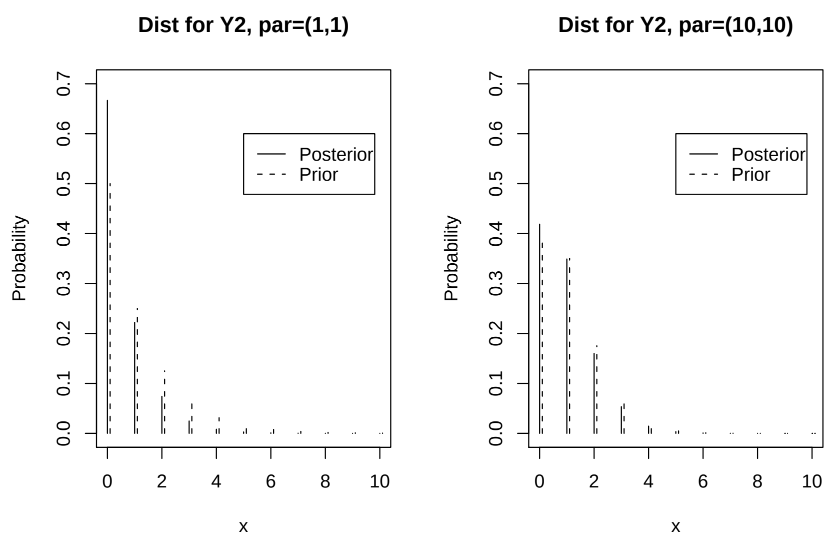


Figure 3: The two figures in the first row show the prior predictive distribution of $f(Y_2)$, and 2nd row shows the posterior predictive distribution $f(Y_2|Y_1 = 0)$ for $Y_2 = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ with $(\alpha, \beta) = (1, 1)$ and $(\alpha, \beta) = (10, 10)$, respectively.



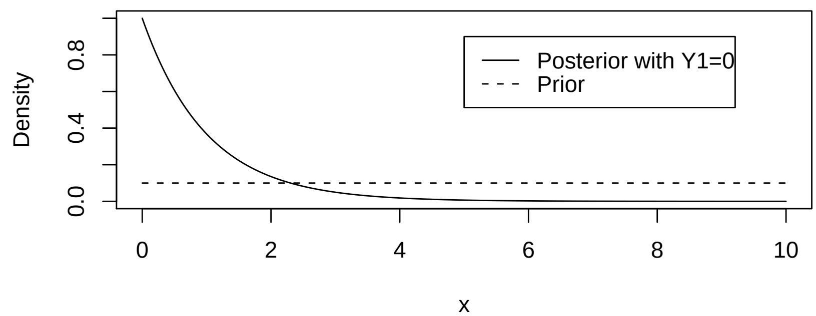
(b)
Now consider another prior distribution for λ : $\lambda \sim U(0,10)$, $f(\lambda) = \frac{1}{10}$, $\lambda \in [0,10]$.

The posterior distribution for $\lambda | Y_1 = y_1$:

$$\begin{aligned} f(\lambda|Y_1 = y_1) &\propto f(Y_1 = y_1|\lambda) f(\lambda) = \frac{1}{y_1!}\lambda^{y_1} e^{-\lambda} \times \frac{1}{10} \\ &\propto \lambda^{y_1} e^{-\lambda}, 0 \le \lambda \le 10. \end{aligned}$$

After observing $Y_1 = 0$, the posterior distribution of $\lambda | Y_1 = 0$ is $e^{-\lambda}/(1 - e^{-10})$.

Figure 4: The solid line shows the prior distribution and The dash line shows the posterior distribution for λ.

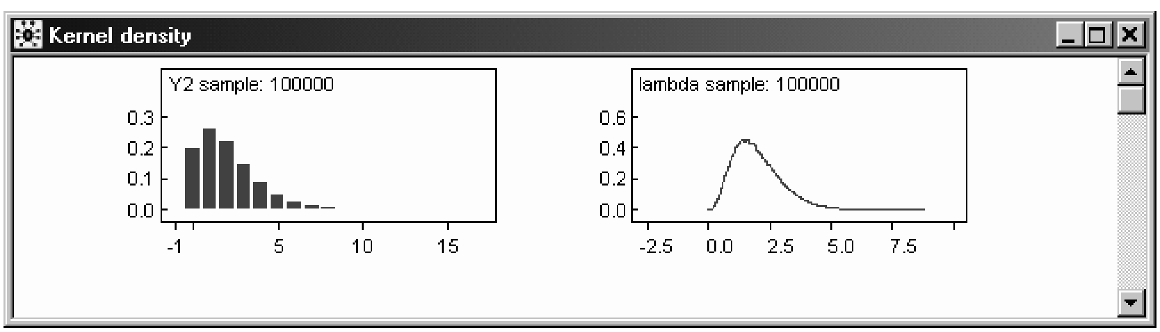


R code for plot:

```
x = seq(0, 10, 0.01)
plot(x,exp(-x)/(1-exp(-10)) , type = "l")
plot(x,rep(1/10,length(x)))
```

■

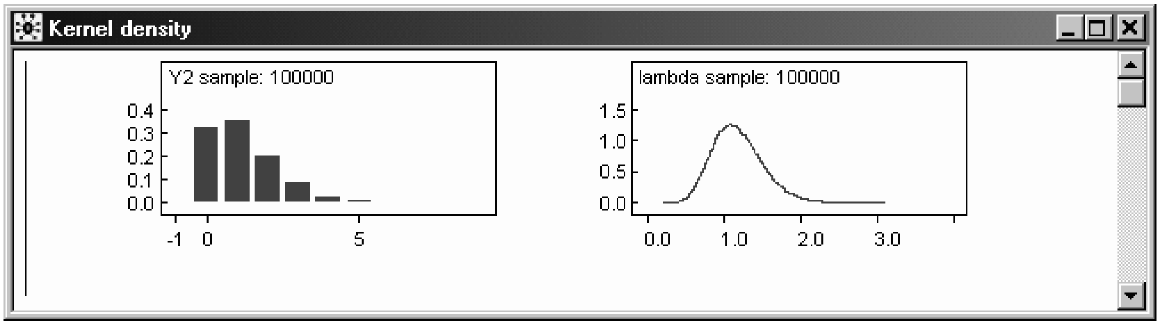
(c) Use WinBUGS to show the approximated posterior distribution.
i. The densities $f(Y_2|Y_1 = 3)$ and $f(\lambda|Y_1 = 3)$, using the gamma prior for λ and $(\alpha, \beta) = (1, 1)$.



Node statistics

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
Y2	2.002	1.734	0.005326	0.0	2.0	6.0	1	100000
lambda	2.002	1.002	0.00314	0.5451	1.838	4.377	1	100000

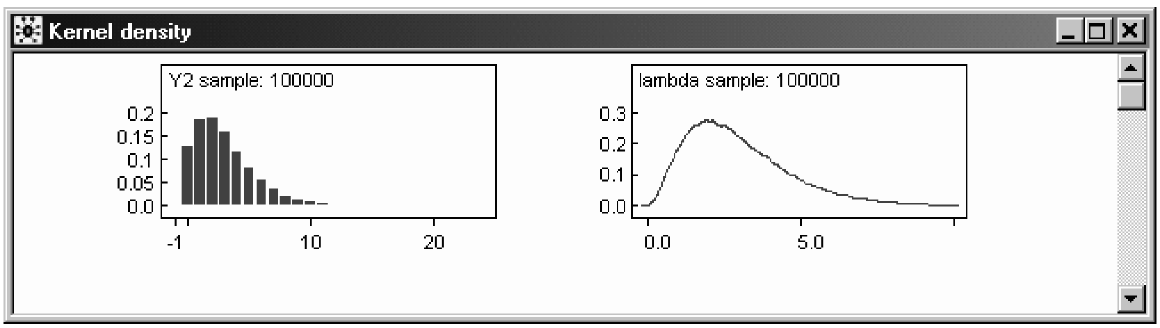
ii. The densities $f(Y_2|Y_1 = 3)$ and $f(\lambda|Y_1 = 3)$, using the gamma prior for λ and $(\alpha, \beta) = (10, 10)$.



Node statistics

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
Y2	1.181	1.134	0.003576	0.0	1.0	4.0	1	100000
lambda	1.182	0.3285	0.001053	0.628	1.152	1.909	1	100000

iii. The density for the posterior $f(Y_2|Y_1 = 3)$ and $f(\lambda|Y_1 = 3)$, using the prior $U(0, 10)$ for λ.

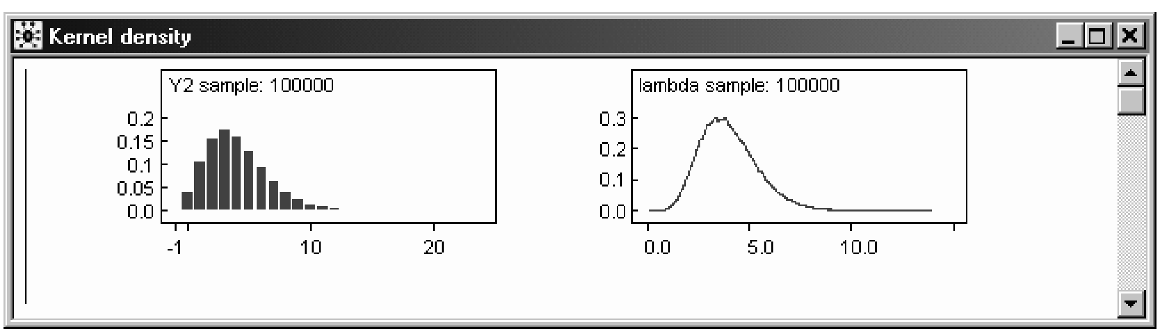


Node statistics

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
Y2	2.985	2.415	0.007741	0.0	2.0	9.0	1	100000
lambda	2.981	1.682	0.005026	0.6218	2.668	7.1	1	100000

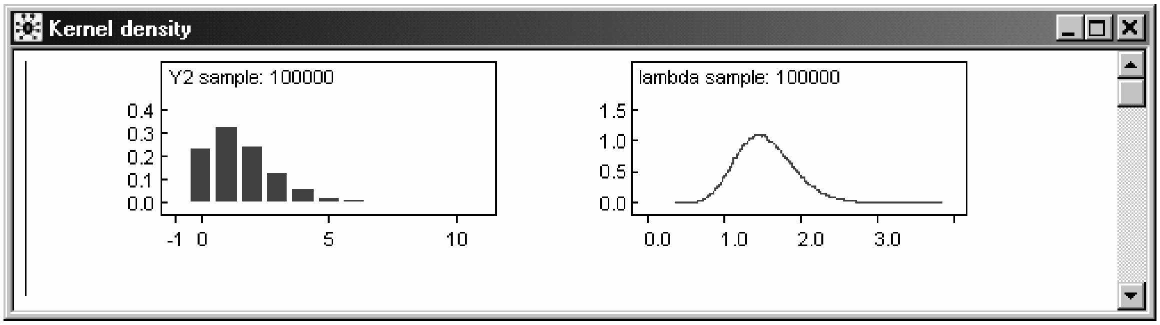
(d)

i. The density for the posterior $f(Y_2|Y_1 = 7)$ and $f(\lambda|Y_1 = 7)$, using the gamma prior for λ and $(\alpha, \beta) = (1, 1)$.



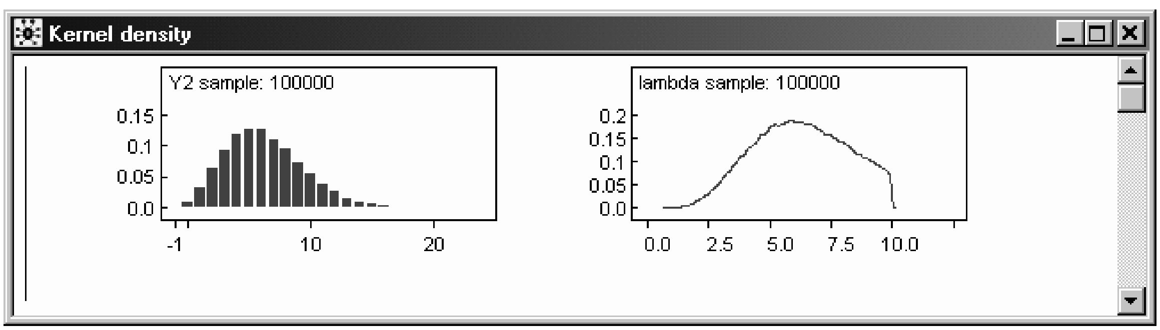
node	mean	sd	MC error	2.5%	median	97.5%	start	sample
Y2	4.006	2.451	0.006953	0.0	4.0	10.0	1	100000
lambda	4.004	1.411	0.004236	1.728	3.841	7.209	1	100000

ii. The density for the posterior $f(Y_2|Y_1 = 7)$ and $f(\lambda|Y_1 = 7)$, using the gamma prior for λ and $(\alpha, \beta) = (10, 10)$.



node	mean	sd	MC error	2.5%	median	97.5%	start	sample
Y2	1.545	1.297	0.003942	0.0	1.0	5.0	1	100000
lambda	1.547	0.3763	0.001159	0.8998	1.517	2.367	1	100000

iii. The density for the posterior $f(Y_2|Y_1 = 7)$ and $f(\lambda|Y_1 = 7)$, using the prior $U(0, 10)$ for λ.



node	mean	sd	MC error	2.5%	median	97.5%	start	sample
Y2	6.281	3.146	0.009689	1.0	6.0	13.0	1	100000
lambda	6.273	1.891	0.006016	2.737	6.248	9.68	1	100000