Stat 543 3-30-05

Recall LRT's

$$\lambda'(x) = \frac{\sup_{\theta \in \Theta} f(x|\theta)}{\sup_{\theta \in \Theta_0} f(x|\theta)}$$

<u>Example</u> $X_1, X_2, \ldots, X_n$ iid $N(\mu, \sigma^2)$

$H_0: \mu = 17$ vs $H_a: \mu \neq 17$

$$\lambda'(x) = \frac{f(x|\hat{\mu}, \hat{\sigma}^2)}{f(x|17, \tilde{\sigma}^2)}$$

$\vdots$

$$\lambda'(x) = \left(\frac{\sum (x_i - 17)^2}{\sum (x_i - \bar{x})^2} \right)^{n/2} \frac{\exp - \frac{n}{2}}{\exp - \frac{n}{2}}$$

rejecting for $\lambda'(x) > k$ is rejecting for

$$\frac{\sum (x_i - \bar{x})^2 + n(\bar{x} - 17)^2}{\sum (x_i - \bar{x})^2} > k'$$

is rejecting for

$$\sqrt{\frac{(\bar{x} - 17)^2}{\sum (x_i - \bar{x})^2}} > k''$$

is rejecting for $$\frac{|\bar{x} - 17|}{\frac{s}{\sqrt{n}}} > k'''$$

i.e. The LRT is the standard t test of $H_0: \mu = 17$ vs $H_a: \mu \neq 17$ as taught in Stat 101 — one choose k" one uses the Stat 542 probability fact that (under H_0)

$$\frac{\bar{X} - 17}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$$

LRT idea is only a heuristic — often it produces a sensible test, but that's not guaranteed

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<u>Example</u> $X \sim$ Exp with mean μ

$H_0: \mu = 7$ vs $H_a: \mu \neq 7$

LRT rejects H_0 if

$$\frac{\sup_{\mu} \frac{1}{\mu} \exp - \frac{x}{\mu}}{\frac{1}{7} \exp - \frac{x}{7}} > k$$

X is the MLE of μ, so this is rejection of H_0 if

$$\frac{7}{x} \exp\left(-1 + \frac{x}{7}\right) > k$$

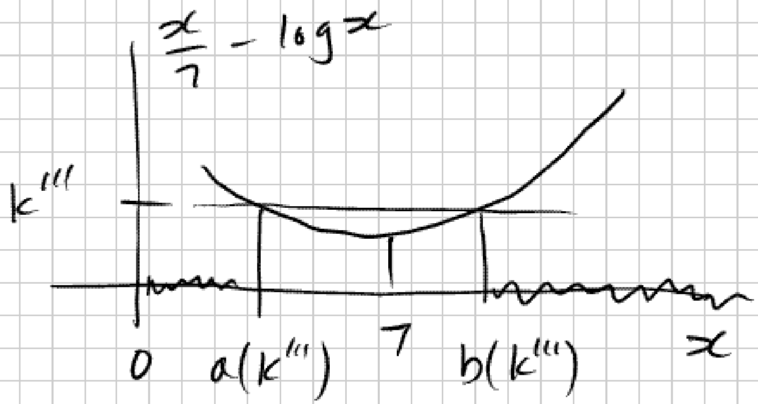
i.e. if

$$\log 7 - \log x - 1 + \frac{x}{7} > k''$$

i.e. if

$$\frac{x}{7} - \log x > k'''$$

Picture



so for a given k''' it is a small numerical problem to find $a(k''') < 7$ and $b(k''') > 7$ solving

$$\frac{x}{7} - \log x = k'''$$

and the CRT is

$$\phi(x) = I\left[x < a(k''') \quad \text{or} \quad x > b(k''') \right]$$

The size of the test is

$$\alpha = P_{\mu=7}\left[\phi(X)=1\right] = 1 - \left(e^{-\frac{a(k''')}{7}} - e^{-\frac{b(k''')}{7}} \right)$$

and by varying k''' we can come up with a test of desired size, α

A bit on Set Estimation and Prediction

Set Estimation $\quad X \sim f(x|\theta) \quad \theta \in \Theta$

$\gamma(\theta)$ a parametric function of interest

Suppose that for each x, $S(x)$ is a subset of $\gamma(\Theta)$, we can call $S(X)$ a (random) set estimator of $\gamma(\theta)$

<u>Set Prediction</u> $\quad (X, Y) \sim f(x, y \mid \theta) \quad \theta \in \Theta$

Suppose that for each x, $S(x)$ is a subset of $\mathcal{Y}$, we can call $S(X)$ a set predictor of Y

set of possible values of Y

<u>Bayes Theory</u> for these problems

$$\theta \sim g(\theta)$$

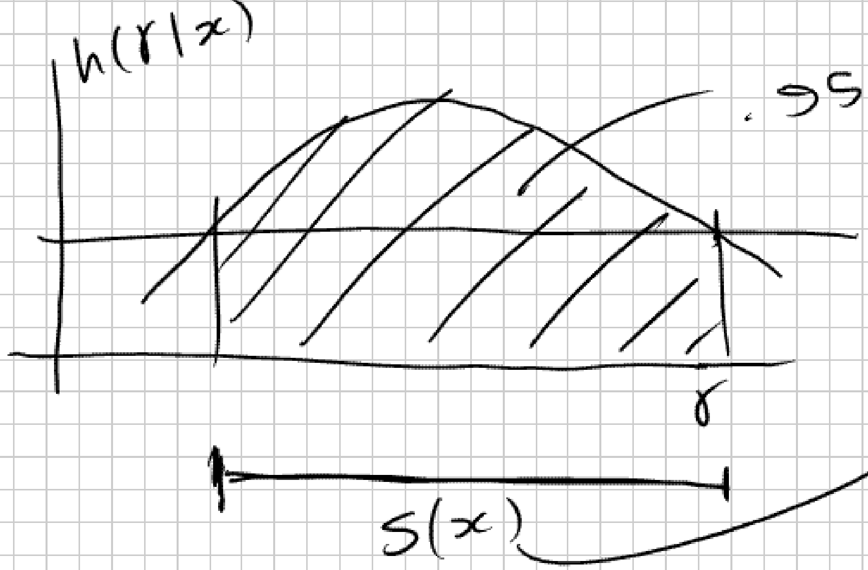
1) this gets me a posterior for θ

$g(\theta|x)$ The posterior for θ

$\Downarrow$

a (posterior) dsn for $\gamma(\theta)$

in the case this dsn is $\text{cont}^{\underline{s}}$ it is common to look for HPD sets for $\gamma(\theta)$



called a 95% HPD (highest posterior density) credible set for $\gamma(\theta)$

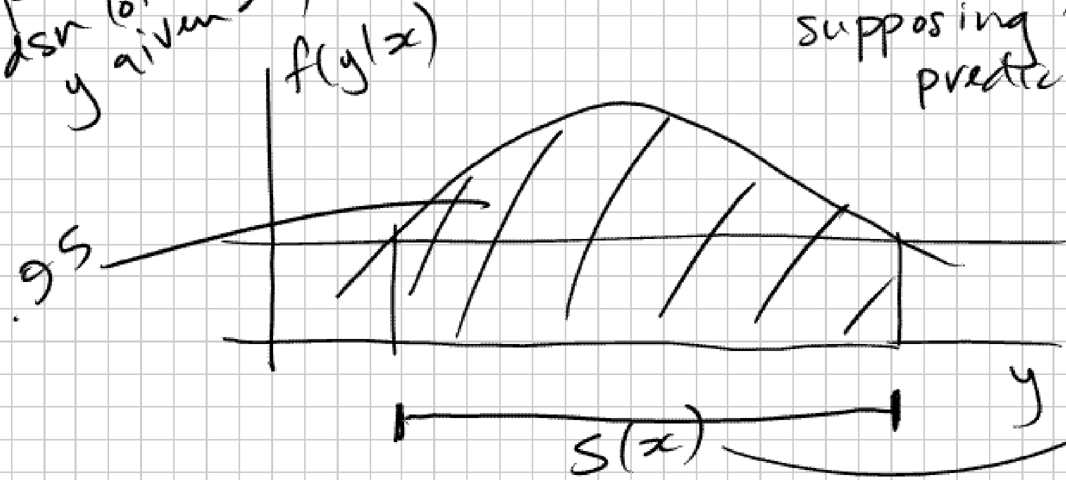
2) for the prediction problem

$$f(x,y\,|\,\theta)\,g(\theta) = f(x,y,\theta)$$

$$f(x,y) = \int f(x,y,\theta)\,d\theta$$

posterior predictive dsn (of y given x) $\rightarrow f(y\,|\,x) = f(x,y)/f(x)$

supposing the posterior predictive dsn is cont's



95% HPD "credible" set for Y