Stat 543 1-10-04

Note Title 1/10/2005

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Syallabus highlights

- 2 evening exams 2/21, 4/4 7-9pm (25% each)
- Final 5/3 7:30 AM (40%)
- HW (10%)

TA Ms. Lu 302 Snedecor

- office Hours Vardeman 10 MWF
  Lu T/R 2 F 1

Text Bickel + Doksum

Read Ch 1

542 Probability The mathematical description of "chance"

Ω sample space
P some (completely specified) probability model
$X(\omega)$ some random quantity

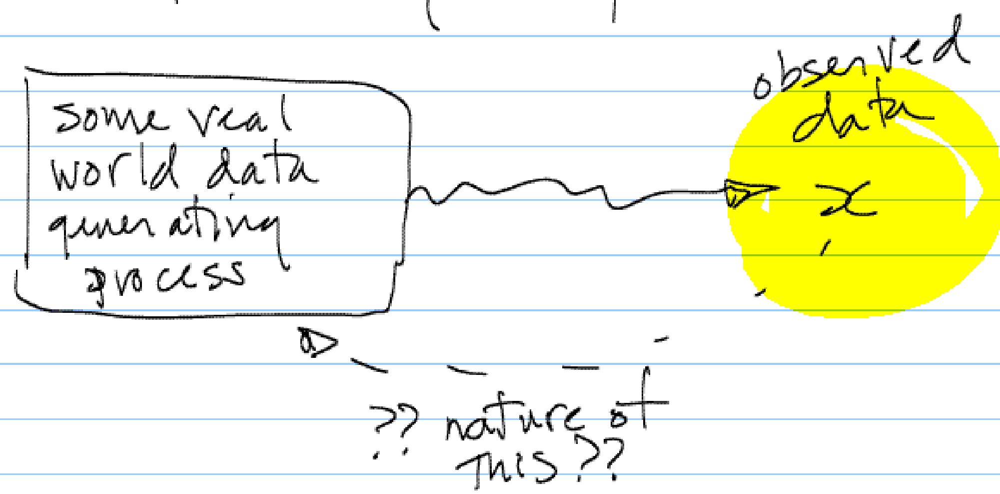
$\Rightarrow$ properties of X, e.g.
$P[X > 13]$ or $P[X_1 > X_2]$
or EX or EX_1X_2

All of this based on P

543 Statistical Theory

The mathematics of rational data analysis

uses probability theory as a tool



Model data generating mechanism as Ω
with some P (an unknown) element of $\mathcal{P}$

Model x as a realized value
of $X(\omega)$

a class of
probability models

often this is taken as a "parametric" family
i.e. $\mathcal{P} = \{ P_\theta \mid \theta \in \Theta \}$ for
$\Theta \subset \mathbb{R}^k$

We will phrase answers to the "what is
the nature of the data generating mechanism?"
in terms of sensible/rational statements
about θ based on x

Example 542

$X_1, X_2, \ldots, X_n$ iid $N(0,1)$ $\Rightarrow$ "P"

e.g. $P\left[\bar{X} > 1.645/\sqrt{n}\right] = .05$

Example 543 some physical process widgets with diameters $4.1, 4.2, 3.8 = x$

$\bar{x} = 4.03$ $s = .21$

what's to be said about widget process on the basis of these data?

Model the data as realizations of $n = 3$ iid $N(\mu, \sigma^2)$ r.v.'s $\theta = (\mu, \sigma)$ $\Theta = \mathbb{R} \times (0, \infty)$

and try to make intelligent/rational/probability-based statements about Θ based on the data

Some standard types of inference are:

1. Estimation (point + set)
2. Testing
3. Prediction

Examples 4.1, 4.2, 3.8 from $N(\mu, \sigma^2)$

1. $\bar{x} = 4.03 = \hat{\mu}$, $s = .21 = \hat{\sigma}$ "point estimates" of μ and σ

$$\left(\bar{x} - t_2 \frac{s}{\sqrt{3}}, \ \bar{x} + t_2 \frac{s}{\sqrt{3}}\right)$$

i.e. $\left(4.03 - t \frac{.21}{\sqrt{3}}, \ 4.03 + t \frac{.21}{\sqrt{3}}\right)$

set estimate (confidence interval for μ)

② For investigate $H_0: \mu = 2$ (with $H_a: \mu > 2$)

$$\frac{\bar{x} - 2}{\frac{s}{\sqrt{3}}} = \frac{4.03 - 2}{\frac{.21}{\sqrt{3}}} = 16.7$$

The observed value of 16.7 makes H_0 look implausible

③ What can be said about the likely value of one additional widget diameter? x_{new}

$$\left(\bar{x} - t_2 s\sqrt{1+\frac{1}{3}} \;,\; \bar{x} - t_2 s\sqrt{1+\frac{1}{3}} \right)$$

i.e. $\left(4.03 - t_2(.21)\sqrt{\frac{4}{3}} \;,\; 4.03 + t_2(.21)\sqrt{\frac{4}{3}} \right)$

is a "prediction interval" for x_{new}