Stat 543 Assignment 8
(not to be collected, but fair game on Exam II, April 4, 2005)
Bayes Tests, Monotone Likelihood Ratio, and UMP Tests

1. If $X \sim$Poisson (λ) and a Bayesian uses a prior for λ that is Exponential with mean 1, what is the Bayesian's (0-1 loss) Bayes test of $\mathrm{H}_0 : \lambda \leq 2$ versus $\mathrm{H}_0 : \lambda > 2$?

2. If $X \sim$Binomial $(5, p)$ and a Bayesian uses a prior for p that is Beta $(2, 1)$, what is the Bayesian's (0-1 loss) Bayes test of $\mathrm{H}_0 : .4 \leq p \leq .6$ versus $\mathrm{H}_0 : p < .4$ or $p > .6$?

3. B&D Problems 4.3.1,4.3.2, 4.3.5, 4.3.6, 4.3.9, 4.3.11

4. (Optional) Prove the following "filling-in" lemma:

   Suppose that g_0 and g_1 are two distinct, positive probability densities defined on an interval in $\mathcal{R}^1$. If the ratio g_1/g_0 is nondecreasing in a real-valued function $T(x)$, then the family of densities $\{g_\alpha |\ \alpha \in [0, 1]\}$ for $g_\alpha = \alpha g_1 + (1 - \alpha) g_2$ has the MLR property in $T(x)$.

5. (Optional) Two possible definitions of "UMP size α" are:

   Definition 1 A test ϕ of H_0:$\theta \in \Theta_0$ vs. H_a:$\theta \in \Theta_1$ is UMP of size α provided

   i) it is of size α, and
   ii) for any other test ϕ' of size α,
   $\pi_\phi(\theta) \geq \pi_{\phi'}(\theta) \quad \forall\, \theta \in \Theta_1$

   Definition 2 ... as in Definition 1, except in ii), let ϕ' be of size $\leq \alpha$

   At first glance, it may seem that Definition 1 is weaker than Definition 2 (it might appear that ϕ could satisfy Definition 1 and fail to satisfy Definition 2). But, in fact, these two definitions are equivalent. Show the equivalence.

   (Hint: If ϕ were to satisfy Definition 1 but not Definition 2, there would need to be a test ϕ' with $\alpha' = \sup_{\theta \in \Theta_0} \pi_{\phi'}(\theta) < \alpha$, such that for some $\theta^* \in \Theta_1$, $\pi_{\phi'}(\theta^*) > \pi_\phi(\theta^*)$. Consider the test $\phi"(x) = \left(\frac{1-\alpha}{1-\alpha'}\right)\phi'(x) + \left(1 - \left(\frac{1-\alpha}{1-\alpha'}\right)\right) 1$.)