STAT542 HW1 SOLUTION

Prob 1.4

a

"either A or B or both" $\Leftrightarrow A \cup B$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

b

"either A or B but not both" $\Leftrightarrow (A \cup B) \backslash (A \cap B) \Leftrightarrow (A \cup B) \cap (A \cap B)^c$

$$\begin{aligned} P((A \cup B) \cap (A \cap B)^c) &= P((A \cup B) \cap (A^c \cup B^c)) \\ &= P((A \cap B^c) \cup P(B \cap A^c) \cup P(A \cap A^c) \cup P(B \cap B^c) \\ &= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] \\ &= P(A) + P(B) - 2P(A \cap B) \end{aligned}$$

c

"at least one of A or B" $\Leftrightarrow A \cup B$. It is the same as (a).

d

"at most one of A or B" $\Leftrightarrow (A \cap B)^c$

$$P((A \cap B)^c) = 1 - P(A \cap B)$$

Prob 1.5

a

$$A \cap B \cap C = \{\text{a U.S. birth results in identical twins that are female}\}$$

b

$$\begin{aligned} P(A \cap B \cap C) &= P(A|B \cap C) \cdot P(B|C) \cdot P(C) \\ &= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{90} \\ &= \frac{1}{540} \end{aligned}$$

Prob 1.13

If A and B are disjoint, then

$$P(A \cup B) = P(A) + P(B) = P(A) + (1 - P(B^c)) = \frac{1}{3} + 1 - \frac{1}{4} = \frac{13}{12} > 1$$

which is impossible. Therefore, A and B cann't be disjoint. More generally, if A and B are disjoint, then $A \subset B^c$ and $P(A) \leq P(B^c)$. But in this case, $P(A) > P(B^C)$. So A and B can't be disjoint.

Prob 1.21

There are totally $\binom{2n}{2r}$ ways of choosing $2r$ shoes from a total of $2n$ shoes. Among them there are $\binom{n}{2r}$ ways of choosing $2r$ different shoe styles. Meanwhile, there are two ways of choosing within a given shoe style (left or right shoe), which gives 2^{2r} ways of arrangment each one of the $\binom{n}{2r}$ arrays. Thus the probability is $\frac{\binom{n}{2r} \cdot 2^{2r}}{\binom{2n}{2r}}$

Prob 1.22

a

Consider the method unordered and without replacement. The probability is

$$\frac{\binom{31}{15}\binom{29}{15}\binom{31}{15}\binom{30}{15}\cdots\binom{31}{15}}{\binom{366}{180}} = \frac{\binom{31}{15}^7\binom{29}{15}\binom{30}{15}^4}{\binom{366}{180}} = 1.67 \times 10^{-9}$$

b

The probability is

$$\frac{\binom{366-30}{30}}{\binom{366}{30}} = \frac{\binom{336}{30}}{\binom{366}{30}} = 0.069$$

Prob 1.33

$$P(CB|M) = 0.05 \quad P(M) = 0.5$$

$$P(CB|W) = 0.0025 \quad P(W) = 0.5$$

Using Bayes rule leads to,

$$\begin{aligned} P(M|CB) &= \frac{P(CB|M)\cdot P(M)}{P(CB|M)\cdot P(M) + P(CB|W)\cdot P(W)} \\ &= \frac{0.05(0.5)}{0.05(0.5) + 0.0025(0.5)} \\ &= 0.9524 \end{aligned}$$

Prob 1.34

a

$$\begin{aligned} P(BH|L1) &= \frac{2}{3} \\ P(GH|L1) &= \frac{1}{3} \\ P(BH|L2) &= \frac{3}{5} \\ P(GH|L2) &= \frac{2}{5} \\ P(L1) &= P(L2) = \frac{1}{2} \\ P(BH) &= P(BH|L1)\cdot P(L1) + P(BH|L2)\cdot P(L2) \\ &= \frac{2}{3}\times\frac{1}{2}\times\frac{3}{5}\times 12 \\ &= \frac{19}{30} \end{aligned}$$

b

$$P(L1|BH) = \frac{P(BH|L1)\cdot P(L1)}{P(BH)} = \frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{19}{30}} = \frac{10}{19}$$

Prob 1.39

a

Suppose A and B are nonempty sets and mutually exclusive, i.e. $A \neq \phi$, $B \neq \phi$, $A \cap B = \phi$ and $P(A \cap B) = 0$. If A and B are independent, then

$$0 = P(A \cap B) = P(A)P(B) \Rightarrow P(A) = 0 \text{ or } P(B) = 0 \Rightarrow A = \phi \text{ or } B = \phi$$

Contradiction. Thus A and B can't be independent.

b

Suppose A and B are nonempty sets and they are independent, i.e. $P(A \cap B) = P(A)P(B) > 0$

$$\Rightarrow A \cap B \neq \phi$$

Thus A snd B are not mutually exclusive.