Stat 542 Exam 1

October 4, 2005 **Prof. Vardeman**

1. Suppose that X is a continuous random variable with pdf

$$f_X(x) = \begin{cases} \frac{1}{8} & \text{if } -2 < x < 0 \\ \frac{3}{8} & \text{if } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

a) Evaluate the mean of X, $\mathrm{E}X$.

b) Let $Y = X^2$. Find a pdf for Y, $f_Y(y)$.

Let

$$W = X \text{ rounded to the nearest integer}$$

(W is a discrete random variable that might be called a "digitized" or "quantized" or "interval censored" version of X .)

c) Find a probability mass function for W , $f_W(w)$. (Record possible values and corresponding probabilities in the table below.)

w	$f_W(w)$

d) How does $\text{Var}\,W$ compare to $\text{Var}\,X = \frac{13}{12}$? (Evaluate $\text{Var}\,W$.)

2. (Moments and MGF's)

a) Argue carefully that there is no distribution for X such that

$$\mathrm{E}\,X = 0,\ \ \mathrm{E}\,X^2 = 1,\ \ \mathrm{E}\,X^3 = 0,\ \text{and}\ \ \mathrm{E}\,X^4 = 0$$

b) Argue carefully that the function $H(t) = 1 + \frac{t^2}{2}$ is not a moment generating function (that there is NO distribution for X for which $M_X(t) = H(t)$ for t in a neighborhood of 0). You may assume the truth of the result in a) whether or not you were able to prove it.

3. A crop scientist studies the laying of eggs by a particular insect species on the leaves of plants of a particular species. Suppose that in a certain field, a plant leaf is k times as likely to be unsuitable for egg laying as it is to be suitable for egg laying. Suppose further that the number of eggs laid on a leaf suitable for egg laying has a $\text{Poisson}(\lambda)$ distribution.

a) A certain leaf is has no eggs on it. What is the conditional probability that the leaf is unsuitable for eggs given this fact?

b) 5 leaves are each found to have eggs on them. What is a reasonable assessment of the (conditional) probability that at least 4 of the 5 each have more than 1 egg on them?

4. A certain security system has passwords that consist of 6 characters. Requirements are that passwords must contain 4 letters (from the 26 letters "a" through "z") with repeats allowed and 2 digits (from the 10 digits "0" through "9"), again with repeats allowed. Passwords are "case-sensitive" so that "a" and "A" are different, and requirements are at least one lower case letter must be employed and at least one capital must be employed. (Of course, order in which the characters appear in the passwords matter.) How many different passwords are there in this system?

5. Suppose $P(A) = .2$, $P(B) = .5$, $P(C) = .4$, and $P(A^c \cap B \cap C) = .04$. Suppose further that A is independent of B and that $A \cap B$ is independent of C. Is B independent of C? Answer yes or no and argue very carefully for the correctness of your answer. (A Venn diagram may help.)