

Summary of Some Useful Facts About Multivariate (Normal) Distributions

1. If X has mean vector μ and covariance matrix Σ , then $Y = B X + d$ has mean vector

$$B \mu + d \text{ and covariance matrix } B \Sigma B'$$

2. The MVN distribution is most usefully defined as the distribution of $X = A Z + \mu$ for Z a vector of independent standard normal random variables. Such a random vector has mean vector μ and covariance matrix $\Sigma = A A'$. (This definition turns out to be unambiguous ... any dimension p and any matrix A giving a particular Σ end up producing the same k -dimensional joint distribution.)

3. If X is $MVN_k(\mu, \Sigma)$ then $Y = B X + d$ is $MVN_l(B \mu + d, B \Sigma B')$.

4. If X is MVN_k , its individual marginal distributions are univariate normal. Further, any sub-vector of dimension $l < k$ is MVN_l (with mean vector the appropriate sub-vector of μ and covariance matrix the appropriate sub-matrix of Σ).

5. If X is $MVN_k(\mu_1, \Sigma_{11})$ and independent of Y which is $MVN_l(\mu_2, \Sigma_{22})$, then the vector

$$W = \begin{pmatrix} X \\ Y \end{pmatrix} \text{ is } MVN_{k+l} \left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix} \right)$$

6. For non-singular Σ the $MVN_k(\mu, \Sigma)$ distribution has a (joint) pdf on k -dimensional space given by

$$f_X(x) = (2\pi)^{-\frac{k}{2}} |\det \Sigma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(x - \mu)' \Sigma^{-1}(x - \mu)\right)$$

7. The joint pdf given in 6 above can be studied and conditional distributions (given values for

part of the X vector) identified. For $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ $MVN_k(\mu, \Sigma)$ where

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

the conditional distribution of X_1 given that $X_2 = x_2$ is MVN_l with

$$\text{mean vector} = \mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (x_2 - \mu_2)$$

and

$$\text{covariance matrix} = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$$

8. All correlations between two parts of a MVN vector equal to 0 implies that those parts of the vector are independent.