Stat 542 Exam 2

November 15, 2001 Prof. Vardeman

1. Below is a table specifying the discrete joint distribution of random variables X and Y.

$y\backslash x$	0	1	2	3	4	5
5	.25	0	0	.01	.03	.02
4	0	0	.01	.03	.02	.03
3	0	.01	.03	.02	.03	.01
2	.01	.03	.02	.03	.01	0
1	.03	.02	.03	.01	0	0
0	.02	.03	.01	0	0	.25

7pts a) Evaluate $P[X+Y>2]$.

7pts b) Evaluate $\mathrm{E}[Y|X=4]$.

7 pts c) Suppose that

$$W = \begin{cases} X+Y & \text{if both } X \text{ and } Y \text{ are odd} \\ 0 & \text{otherwise} \end{cases}$$

Write out a sum that is $\mathrm{E}W$. (You need not do the arithmetic necessary to simplify this.)

2. (Moore) Suppose that in appropriate units, the following is true. The length of a standard bar of steel is μ. A copy of the bar is not perfect, and has length L_1 so that $D_1 = L_1 - \mu \sim \mathrm{N}(0,1)$. A copy of this copy has length L_2 so that $D_2 = L_2 - L_1 \sim \mathrm{N}(0,1)$ and D_2 is independent of L_1. A copy of the copy of the copy has length L_3 so that $D_3 = L_3 - L_2 \sim \mathrm{N}(0,1)$ and D_3 is independent of L_1 and L_2.

10 pts a) What is the (joint) distribution of $\boldsymbol{L} = (L_1, L_2, L_3)'$?

5 pts b) Evaluate the correlation between L_1 and L_3.

5 pts c) What is the distribution of $L_3 - \mu$, the error in length of the last bar?

3. Suppose that $U \sim$ Uniform $(0,1)$ and that the conditional distribution of $Y|U = u$ is $\mathrm{N}(u,1)$.

7pts a) Evaluate Var$Y$.

7 pts b) Evaluate Cov(U, Y).

4. Suppose that X and Y are jointly "uniform on the unit circle," i.e. jointly continuous with pdf

$$f(x,y) = \begin{cases} \frac{1}{\pi} & \text{if } \ x^2 + y^2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

7 pts a) For $x \in (-1,1)$, what is the conditional distribution of $Y|X = x$?

5 pts b) Are X and Y independent? Explain.

20 pts 5. Find a pdf for $S = \frac{Z}{U}$ where Z and U are independent random variables, $Z \sim \mathrm{N}(0,1)$ and $U \sim \text{Uniform}\,(0,1)$. (Among other possibilities, a transformation will work here.)

6. The Poisson (λ) moment generating function is

$$M(t) = \exp(\lambda(\exp(t) - 1))$$

7 pts (a) If $X \sim$ Poisson (λ), what is the mgf of $Z = (X - \lambda)/\sqrt{\lambda}$, say $M_Z(t)$?

6 pts (b) A second order Taylor expansion implies that $\exp(x) = 1 + x + \frac{x^2}{2} + R(x)$ where $|R(x)| \leq \frac{\exp(|x|)|x|^3}{6}$. Apply this to the form you found in part (a) and identify a limit for $M_Z(t)$ as $\lambda \to \infty$. (Don't approximate the "outside exponential," only its argument.)