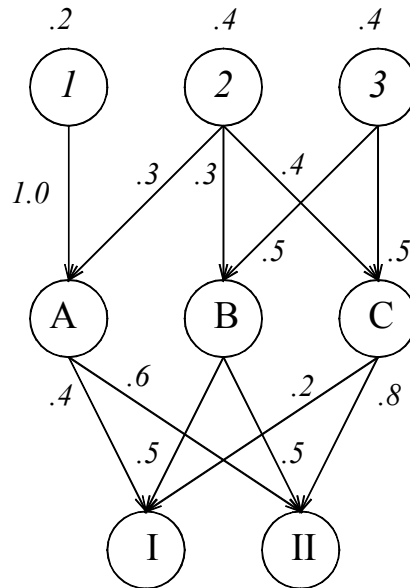


# Stat 542 Exam I

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1. A rat enters a maze at one of the nodes 1, 2 and 3, proceeds through one of nodes A, B and C, to end at one of nodes I and II. Below is a schematic of the maze with (conditional) probabilities marked on it indicating probabilities of taking various next paths, given the rat reaches a particular node. (The values at the top of the diagram indicate the probabilities of beginning at nodes 1, 2 and 3.)



7 pts a) Write out (but you need not evaluate) a numerical expression for the probability that a rat ends at node II.

8 pts b) Write out (but you need not evaluate) a numerical expression for the conditional probability that a rat began at node 2, given that it ends at node II.

2. A random variable  $X$  has pdf

$$f(x) = \begin{cases} 4x & 0 < x < \frac{1}{2} \\ 4(1-x) & \frac{1}{2} < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Carefully determine the following:

6 pts a)  $EX$

6 pts b)  $\text{Var}X$

6 pts c)  $F(x)$

6 pts d) A pdf for the random variable  $Y = -\ln(X)$

3. A famous (and incredibly popular) statistician has 8 shirts, 5 of which are blue. He wears a clean one each day of the week, does laundry on the weekends and begins each Monday morning with a closet full of 8 clean shirts. He teaches a course that meets 4 days per week.

Suppose that this person actually chooses which shirt he wears on a given day at random from those remaining in his closet.

- 7 pts a) What is the probability that in a particular week of 4 lectures, his students see only blue shirts?
- 9 pts b) What is the probability that in a particular week of 4 lectures, the students see their 2nd blue shirt on day 4?
- 6 pts c) What is the probability that during the first 6 weeks of class, the students see only blue shirts?
- 9 pts 4. Suppose that  $U \sim \text{Uniform}(0, 1)$  and that  $F$  is a continuous, strictly increasing cdf (so that  $F$  has an inverse,  $F^{-1}$ ). What is the distribution of  $Y = F^{-1}(U)$ ? Argue carefully for your answer.

8 pts 5. Suppose that  $X \sim \text{Poisson}(\lambda)$ . Evaluate  $E\left(\frac{1}{X+1}\right)$ .

6. Suppose that  $Z \sim \text{Normal}(0, 1)$ . I know something about the realized value of  $Z$  and tell you that  $Z > -1$ .

6 pts a) What do you propose to use as a cdf for  $Z$ , *conditional on the information that  $Z > -1$* ? (What is  $P[Z \leq z | Z > -1]$ ? Your answer may be in terms of values of  $\Phi(z)$ .)

6 pts b) Write out an explicit expression for a sensible "mean value of  $Z$  conditioned on  $Z > -1$ " (but don't bother to evaluate this). (Your answer may be in terms of  $\Phi(z)$  and  $\phi(z)$ .)

10 pts 7. Find an expression for  $\text{Var}(3X(X - 1) + 5)$  in terms of moments of  $X$ ,  $EX^r$ ,  $r = 1, 2, 3, 4$ .